Production-decline analysis is the analysis of past trends of declining production performance, that is, rate versus time and rate versus cumulative production plots, for wells and reservoirs. From about 1975 to 2005, various methods were developed for estimating reserves in tight gas reservoirs. These methods range from the basic material balance equation to decline- and type-curve analysis techniques. There are two kinds of decline-curve analysis techniques, namely,

• The classical curve fit of historical production data
• The type-curve matching technique

Some graphical solutions use a combination of decline curves and type curves with varying limitations. General principles of both types and methods of combining both approaches to determine gas reserves are briefly presented in this chapter.

DECLINE-CURVE ANALYSIS

Decline curves are one of the most extensively used forms of data analysis employed in evaluating gas reserves and predicting future production. The decline-curve analysis technique is based on the assumption that past production trends and their controlling factors will continue in the future and, therefore, can be extrapolated and described by a mathematical expression.
The method of extrapolating a “trend” for the purpose of estimating future performance must satisfy the condition that the factors that caused changes in past performance, for example, decline in the flow rate, will operate in the same way in the future. These decline curves are characterized by three factors:

- Initial production rate, or the rate at some particular time
- Curvature of the decline
- Rate of decline

These factors are a complex function of numerous parameters within the reservoir, wellbore, and surface-handling facilities.

Ikoku (1984) presented a comprehensive and rigorous treatment of production-decline-curve analysis. He pointed out that the following three conditions must be considered in production-decline-curve analysis:

1. Certain conditions must prevail before we can analyze a production-decline curve with any degree of reliability. The production must have been stable over the period being analyzed; that is, a flowing well must have been produced with constant choke size or constant wellhead pressure and a pumping well must have been pumped off or produced with constant fluid level. These indicate that the well must have been produced at capacity under a given set of conditions. The production decline observed should truly reflect reservoir productivity and not be the result of an external cause, such as a change in production conditions, well damage, production controls, or equipment failure.

2. Stable reservoir conditions must also prevail in order to extrapolate decline curves with any degree of reliability. This condition will normally be met as long as the producing mechanism is not altered. However, when an action is taken to improve the recovery of gas, such as infill drilling, fluid injection, fracturing, or acidizing, decline-curve analysis can be used to estimate the performance of the well or reservoir in the absence of the change and compare it to the actual performance with the change. This comparison will enable us to determine the technical and economic success of our efforts.

3. Production-decline-curve analysis is used in the evaluation of new investments and the audit of previous expenditures. Associated with this is the sizing of equipment and facilities such as pipelines, plants, and treating facilities. Also associated with the economic analysis is the determination of reserves for a well, lease, or field. This is an inde-
pendent method of reserve estimation, the result of which can be com-
pared to volumetric or material-balance estimates.

Arps (1945) proposed that the “curvature” in the production-rate-ver-
sus-time curve can be expressed mathematically by a member of the
hyperbolic family of equations. Arps recognized the following three
types of rate-decline behavior:

- Exponential decline
- Harmonic decline
- Hyperbolic decline

Each type of decline curve has a different curvature, as shown in Fig-
ure 16-1. This figure depicts the characteristic shape of each type of
decline when the flow rate is plotted versus time or versus cumulative
production on Cartesian, semi log, and log-log scales. The main charac-
teristics of these decline curves can be used to select the flow-rate-
decline model that is appropriate for describing the rate–time relationship
of the hydrocarbon system:

- **For exponential decline**: A straight-line relationship will result when
the flow rate versus time is plotted on a semi log scale and also when

---

**Figure 16-1.** Classification of production decline curves. (After Arps, J.J. “Estima-
the flow rate versus cumulative production is plotted on a Cartesian scale.

- **For harmonic decline:** Rate versus cumulative production is a straight line on a semi log scale; all other types of decline curves have some curvature. There are several shifting techniques that are designed to straighten out the curve that results from plotting flow rate versus time on a log-log scale.

- **For hyperbolic decline:** None of the above plotting scales, that is, Cartesian, semi log, or log-log, will produce a straight-line relationship for a hyperbolic decline. However, if the flow rate is plotted versus time on log-log paper, the resulting curve can be straightened out with shifting techniques.

Nearly all conventional decline-curve analysis is based on empirical relationships of production rate versus time, given by Arps (1945) as follows:

\[
q_t = \frac{q_i}{(1 + bD_i t)^{1/b}}
\]  

(16-1)

where
- \(q_t\) = gas flow rate at time \(t\), MMscf/day
- \(q_i\) = initial gas flow rate, MMscf/day
- \(t\) = time, days
- \(D_i\) = initial decline rate, day \(^{-1}\)
- \(b\) = Arps’ decline-curve exponent

The mathematical description of these production-decline curves is greatly simplified by the use of the instantaneous (nominal) decline rate, \(D\). This decline rate is defined as the rate of change of the natural logarithm of the production rate, that is, \(\ln(q)\), with respect to time, \(t\), or

\[
D = -\frac{d (\ln q)}{dt} = -\frac{1}{q} \frac{dq}{dt}
\]  

(16-2)

The minus sign has been added because \(dq\) and \(dt\) have opposite signs and it is convenient to have \(D\) always positive. Notice that the decline-rate equation, Equation 16-2, describes the instantaneous changes in the slope of the curvature, \(dq/dt\), with the change in the flow rate, \(q\), over time.
The parameters determined from the classical fit of the historical data, namely the decline rate, $D$, and the exponent, $b$, can be used to predict future production. This type of decline-curve analysis can be applied to **individual wells or the entire reservoir**. The accuracy of the entire-reservoir application is sometimes even better than for individual wells due to smoothing of the rate data. Based on the type of rate-decline behavior of the hydrocarbon system, the value of $b$ ranges from 0 to 1, and, accordingly, Arps’ equation can be conveniently expressed in the following three forms:

<table>
<thead>
<tr>
<th>Case</th>
<th>$b$</th>
<th>Rate–Time Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>$b = 0$</td>
<td>$q_t = q_i \exp(-D_i t)$</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>$0 &lt; b &lt; 1$</td>
<td>$q_t = \frac{q_i}{(1 + b D_i t)^{1/b}}$</td>
</tr>
<tr>
<td>Harmonic</td>
<td>$b = 1$</td>
<td>$q_t = \frac{q_i}{(1 + D_i t)}$</td>
</tr>
</tbody>
</table>

Figure 16-2 illustrates the general shape of the three curves at different possible values of $b$. These mathematical relations can be applied equally for gas and oil reservoirs.

It should be pointed out that these three forms of decline-curve equations are **applicable ONLY when the well/reservoir is under pseudosteady (semi steady)-state flow conditions, that is, boundary-dominated flow conditions**. Arps’ equation has been often misused to model the performance of oil and gas wells whose flow regimes are in a transient state. As presented in Chapter 6, when a well is first open to flow, it is in a transient (unsteady-state) condition. It remains in this condition until the production from the well affects the total reservoir system by reaching its drainage boundary, at which time the well is said to be flowing in a pseudosteady-state or **boundary-dominated flow condition**. The following is a list of inherent assumptions that must be satisfied before performance of rate–time decline-curve analysis:

- The well is draining a constant drainage area, that is, the well is in a boundary-dominated flow condition.
The well is produced at or near capacity.

The well is produced at a constant bottom-hole pressure.

Again, these three conditions must be satisfied before any of the decline-curve analysis methods is applied to describe the production performance of a reservoir. In most cases, tight gas wells are producing at capacity and approach a constant bottom-hole pressure if produced at a constant line pressure. However, it can be extremely difficult to determine when a tight gas well has defined its drainage area and thus to identify the start of the pseudosteady-state flow condition.

The area under the decline curve of q versus time between the times \( t_1 \) and \( t_2 \) is a measure of the cumulative oil or gas production during this period. Dealing with gas reservoirs, the cumulative gas production, \( G_p \), can be expressed mathematically:

\[
G_p = \int_{t_1}^{t_2} q_i \, dt
\]  

(16-6)
Replacing the flow rate, \( q_t \), in the above equation with the three individual expressions that describe types of decline curves (Equations 16-3, 16-4, and 16-5), and integrating gives the following:

**Exponential** \( b = 0 \): 
\[
G_{p(t)} = \frac{q_i - q_t}{D_i} 
\]  
(16-7)

**Hyperbolic** \( 0 < b < 1 \): 
\[
G_{p(t)} = \left[ \frac{q_i}{D_i(1-b)} \right] \left[ 1 - \left( \frac{q_t}{q_i} \right)^{1-b} \right] 
\]  
(16-8)

**Harmonic** \( b = 1 \): 
\[
G_{p(t)} = \left( \frac{q_i}{D_i} \right) \ln \left( \frac{q_t}{q_i} \right) 
\]  
(16-9)

where \( G_{p(t)} \) = cumulative gas production at time \( t \), MMscf
\( q_i \) = initial gas flow rate at time \( t = 0 \), MMscf/unit time
\( t \) = time, unit time
\( q_t \) = gas flow rate at time \( t \), MMscf/unit time
\( D_i \) = nominal (initial) decline rate, 1/unit time

All the expressions given by Equations 16-3 through 16-9 require **consistent units**. Any convenient unit of time can be used, but, again, care should be taken to make certain that the time unit of the gas flow rates, \( q_i \) and \( q_t \), matches the time unit of the decline rate, \( D_i \), for example, for flow rate \( q \) in scf/month or STB/month with \( D_i \) in month\(^{-1}\).

Note that the traditional Arps decline-curve analysis, as given in Equations 16-7 through 16-9, gives a reasonable estimation of reserve but also has its failings, the most important one being that it **completely ignores the flowing pressure data**. As a result, it can underestimate or overestimate the reserves. The practical applications of these three commonly used decline curves for gas reservoirs are as follows:

**Exponential Decline, \( b = 0 \)**

The graphical presentation of this type of decline curve indicates that a plot of \( q_t \) versus \( t \) on a semi log scale or a plot of \( q_t \) versus \( G_{p(t)} \) on a Cartesian scale will produce linear relationships that can be described mathematically by
Similarly,

\[ G_{p(t)} = \frac{q_t - q_i}{D_i} \]

or linearly as

\[ q_t = q_i - D_i G_{p(t)} \]

This type of decline curve is perhaps the simplest to use and perhaps the most conservative. It is widely used in the industry for the following reasons:

- Many wells follow a constant decline rate over a great portion of their productive life and will deviate significantly from this trend toward the end of this period.
- The mathematics involved, as described by the line expressions just given, are easier to apply than those for the other line types.

Assuming that the historical production from a well or field is recognized by its exponential production-decline behavior, the following steps summarize the procedure to predict the behavior of the well or the field as a function of time.

**Step 1.** Plot \( q_t \) versus \( G_p \) on a Cartesian scale and \( q_t \) versus \( t \) on semi-log paper.

**Step 2.** For both plots, draw the best straight line through the points.

**Step 3.** Extrapolate the straight line on \( q_t \) versus \( G_p \) to \( G_p = 0 \), which intercepts the y-axis with a flow rate value that is identified as \( q_i \).

**Step 4.** Calculate the initial decline rate, \( D_i \), by selecting a point on the Cartesian straight line with a coordinate of \( (q_t, G_p) \) or on a semi-log line with a coordinate of \( (q_t, t) \) and solve for \( D_i \) by applying Equation 16-5 or Equation 16-7.
or equivalently as

\[ D_i = \frac{q_i - q_i}{G_{p(t)}} \quad (16-11) \]

If the method of least squares is used to determine the decline rate by analyzing all of the production data, then

\[ D_i = \frac{\sum t \ln(q_i/q_t)}{\sum t^2} \quad (16-12) \]

or equivalently as

\[ D_i = \frac{n \sum (q_t G_{p(t)}) - \left[ \left( \sum q_t \right) \left( \sum G_{p(t)} \right) \right]}{n \sum (G_{p(t)})^2 - \left( \sum G_{p(t)} \right)^2} \quad (16-13) \]

where \( n \) is the number of data points.

**Step 5.** Calculate the time it will take to reach the economic flow rate, \( q_a \) (or any rate) and corresponding cumulative gas production from Equations 16-3 and 16-7.

\[ t_a = \frac{\ln(q_i/q_a)}{D_i} \]

\[ G_{pa} = \frac{q_i - q_a}{t_a} \]

where \( G_{pa} = \) cumulative gas production when reaching the economic flow rate or at abandonment, MMscf

\( q_i = \) initial gas flow rate at time \( t = 0 \), MMscf/unit time
Example 16-1

The following production data are available from a dry gas field:

<table>
<thead>
<tr>
<th>$q_t$, MMscf/day</th>
<th>$G_p$, MMscf</th>
<th>$q_t$, MMscf/day</th>
<th>$G_p$, MMscf</th>
</tr>
</thead>
<tbody>
<tr>
<td>320</td>
<td>16,000</td>
<td>208</td>
<td>304,000</td>
</tr>
<tr>
<td>336</td>
<td>32,000</td>
<td>197</td>
<td>352,000</td>
</tr>
<tr>
<td>304</td>
<td>48,000</td>
<td>184</td>
<td>368,000</td>
</tr>
<tr>
<td>309</td>
<td>96,000</td>
<td>176</td>
<td>384,000</td>
</tr>
<tr>
<td>272</td>
<td>160,000</td>
<td>184</td>
<td>400,000</td>
</tr>
<tr>
<td>248</td>
<td>240,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimate
(a) The future cumulative gas production when the gas flow rate reaches 80 MMscf/day
(b) Extra time to reach 80 MMscf/day

Solution

Part a

Step 1. A plot of $G_p$ versus $q_t$ on a Cartesian scale, as shown in Figure 16-3, produces a straight line indicating an exponential decline.

Step 2. From the graph, cumulative gas production is 633,600 MMscf at $q_t = 80$ MMscf/day, indicating an extra production of $633.6 - 400.0 = 233.6$ MMMscf.

Step 3. The intercept of the straight line with the y-axis gives a value of $q_i = 344$ MMscf/day.

Step 4. Calculate the initial (nominal) decline rate $D_i$ by selecting a point on the straight line and solving for $D_i$ by applying Equation 16-11. Selecting a $G_{p(t)}$ of 352 MMscf, at a $q_t$ of 197 MMscf/day, gives

$$D_i = \frac{q_i - q_t}{G_{p(t)}} = \frac{344 - 197}{352,000} = 0.000418 \text{ day}^{-1}$$
It should be pointed out that the monthly and yearly nominal decline, that is, $D_{\text{im}}$ and $D_{\text{iy}}$, respectively, can be determined as

$$D_{\text{im}} = (0.000418) (30.4) = 0.0126 \text{ month}^{-1}$$

$$D_{\text{iy}} = (0.0126) (12) = 0.152 \text{ year}^{-1}$$

Using the least-squares approach from Equation 16-13 gives

$$D_i = \frac{n \sum_t (q_t G_{p(t)}) - \left[ \left( \sum_t q_t \right) \left( \sum_t G_{p(t)} \right) \right]}{n \sum_t (G_{p(t)})^2 - \left( \sum_t G_{p(t)} \right)^2}$$

$$D_i = \frac{5.55104(10^9) - 6.5712(10^9)}{8.3072(10^{12}) - 5.760(10^{12})} = 0.000401 \text{ day}^{-1}$$

**Part b**

To calculate the extra time to reach 80 MMscf/day, apply the following steps:
Step 1. Calculate the time to reach the last recorded flow rate, 184 MMscf, using Equation 16-10:

\[
t_a = \frac{\ln(q_i/q_{a})}{D_i} = \frac{\ln(344/184)}{0.000401} = 1560 \text{ days} = 4.275 \text{ year}
\]

Step 2. Calculate the total time to reach a gas flow rate of 80 MMscf/day:

\[
t = \frac{\ln(344/80)}{0.000401} = 3637 \text{ days} = 9.966 \text{ years}
\]

Step 3. Extra time = 9.966 – 4.275 – 5.691 years

Example 16-2

A gas well has the following production history:

<table>
<thead>
<tr>
<th>Date</th>
<th>Time t, months</th>
<th>( q_t, ) MMscf/month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1-02</td>
<td>0</td>
<td>1240</td>
</tr>
<tr>
<td>2-1-02</td>
<td>1</td>
<td>1193</td>
</tr>
<tr>
<td>3-1-02</td>
<td>2</td>
<td>1148</td>
</tr>
<tr>
<td>4-1-02</td>
<td>3</td>
<td>1104</td>
</tr>
<tr>
<td>5-1-02</td>
<td>4</td>
<td>1066</td>
</tr>
<tr>
<td>6-1-02</td>
<td>5</td>
<td>1023</td>
</tr>
<tr>
<td>7-1-02</td>
<td>6</td>
<td>986</td>
</tr>
<tr>
<td>8-1-02</td>
<td>7</td>
<td>949</td>
</tr>
<tr>
<td>9-1-02</td>
<td>8</td>
<td>911</td>
</tr>
<tr>
<td>10-1-02</td>
<td>9</td>
<td>880</td>
</tr>
<tr>
<td>11-1-02</td>
<td>10</td>
<td>843</td>
</tr>
<tr>
<td>12-1-02</td>
<td>11</td>
<td>813</td>
</tr>
<tr>
<td>1-1-03</td>
<td>12</td>
<td>782</td>
</tr>
</tbody>
</table>

(a) Use the first six months of the production history data to determine the coefficient of the decline-curve equation.
(b) Predict flow rates and cumulative gas production from August 1, 2002 through January 1, 2003.
(c) Assuming that the economic limit is 30 MMscf/month, estimate the time to reach the economic limit and the corresponding cumulative gas production.
Solution

Part a

Step 1. A plot of $q_t$ versus $t$ on a semi log scale, as shown in Figure 16-4, indicates an exponential decline.

Step 2. Determine the initial decline rate, $D_i$, by selecting a point on the straight line and substituting the coordinates of the point in Equation 16-10 to give

$$D_i = \frac{\ln(q_i/q_t)}{t} = \frac{\ln(1240/986)}{6} = 0.0382 \text{ month}^{-1}$$

Alternatively, using the least-squares method as expressed by Equation 16-12 gives

$$D_i = \frac{\sum t \ln(q_i/q_t)}{\sum t^2}$$

![Figure 16-4. Decline-curve data for Example 16-2.]
Part b

Use Equations 16-3 and 16-7 to calculate $q_t$ and $G_{p(t)}$, and tabulate the results as follows:

$$q_t = q_i \exp(-D_i t) = 1240 \exp(-0.0383 t)$$

$$G_{p(t)} = \frac{(q_i - q_t)}{D_i} = \frac{(q_i - q_t)}{0.0383}$$

<table>
<thead>
<tr>
<th>Date</th>
<th>Time, months</th>
<th>Actual $q_i$, MMscf/month</th>
<th>Calculated $q_i$, MMscf/month</th>
<th>$G_{p(t)}$, MMscf/month</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1-02</td>
<td>1</td>
<td>1193</td>
<td>1193</td>
<td>1217</td>
</tr>
<tr>
<td>3-1-02</td>
<td>2</td>
<td>1148</td>
<td>1149</td>
<td>2387</td>
</tr>
<tr>
<td>4-1-02</td>
<td>3</td>
<td>1104</td>
<td>1105</td>
<td>3514</td>
</tr>
<tr>
<td>5-1-02</td>
<td>4</td>
<td>1066</td>
<td>1064</td>
<td>4599</td>
</tr>
<tr>
<td>6-1-02</td>
<td>5</td>
<td>1023</td>
<td>1026</td>
<td>4643</td>
</tr>
<tr>
<td>7-1-02</td>
<td>6</td>
<td>986</td>
<td>986</td>
<td>6647</td>
</tr>
<tr>
<td>8-1-02</td>
<td>7</td>
<td>949</td>
<td>949</td>
<td>7614</td>
</tr>
<tr>
<td>9-1-02</td>
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<td>911</td>
<td>913</td>
<td>8545</td>
</tr>
<tr>
<td>10-1-02</td>
<td>9</td>
<td>880</td>
<td>879</td>
<td>9441</td>
</tr>
<tr>
<td>11-1-02</td>
<td>10</td>
<td>843</td>
<td>846</td>
<td>10,303</td>
</tr>
<tr>
<td>12-1-02</td>
<td>11</td>
<td>813</td>
<td>814</td>
<td>11,132</td>
</tr>
<tr>
<td>1-1-03</td>
<td>12</td>
<td>782</td>
<td>783</td>
<td>11,931</td>
</tr>
</tbody>
</table>

Part c

Apply Equations 16-10 and 16-11 to calculate the time, $t_a$, to reach an economic flow rate, $q_a$, of 30 MMscf/month, and the corresponding reserves, $G_{pa}$:

$$t_a = \frac{\ln(q_i/q_a)}{D_i} = \frac{\ln(1240/30)}{0.0383} = 97 \text{ months} = 8 \text{ year}$$

$$G_{pa} = \frac{q_i - q_a}{t_a} = \frac{(1240 - 30)10^6}{0.0383} = 31.6 \text{ MMscf}$$
Harmonic Decline, \( b = 1 \)

The production-recovery performance of a hydrocarbon system that follows a harmonic decline (i.e., \( b = 1 \) in Equation 16-1) is described by Equations 16-5 and 16-9.

\[
q_i = \frac{q_i}{1 + D_i t}
\]

\[
G_{p(t)} = \left( \frac{q_i}{D_i} \right) \ln \left( \frac{q_i}{q_i} \right)
\]

These two expressions can be rearranged and expressed as follows:

\[
\frac{1}{q_i} = \frac{1}{q_i} + \left( \frac{D_i}{q_i} \right) t \quad (16-14)
\]

\[
\ln(q_i) = \ln(q_i) - \left( \frac{D_i}{q_i} \right) G_{p(t)} \quad (16-15)
\]

The basic two plots for harmonic decline-curve analysis are based on these two relationships. Equation 16-14 indicates that a plot of \( 1/q_i \) versus \( t \) on a Cartesian scale will yield a straight line with a slope of \( (D_i/q_i) \) and an intercept of \( 1/q_i \). Equation 16-15 suggests a plot of \( q_i \) versus \( G_{p(t)} \) on a semi log scale and will yield a straight line with a negative slope of \( -D_i/q_i \) and an intercept of \( q_i \). The method of least squares can also be used to calculate the decline rate, \( D_i \), to give

\[
D_i = \frac{\sum \left( \frac{t q_i}{q_i} \right) - \sum t}{\sum t^2}
\]

Other relationships that can be derived from Equations 16-14 and 16-15 include the time to reach the economic flow rate, \( q_a \) (or any flow rate), and the corresponding cumulative gas production, \( G_{p(a)} \):
Hyperbolic Decline, 0 < b < 1

The two governing relationships for a reservoir or a well whose production follows the hyperbolic decline behavior are given by Equations 16-4 and 16-8:

\[ q_t = \frac{q_i}{(1 + b D_i t)^{1/b}} \]

\[ G_{p(t)} = \left[ \frac{q_i}{D_i (1 - b)} \right] \ln \left( \frac{q_i}{q_t} \right) \]

The following simplified iterative method is designed to determine \( D_i \) and \( b \) from the historical production data.

**Step 1.** Plot \( q_t \) versus \( t \) on a semi log scale and draw a smooth curve through the points.

**Step 2.** Extend the curve to intercept the \( y \)-axis at \( t = 0 \) and read \( q_i \).

**Step 3.** Select the other end-point of the smooth curve, record the coordinates of the point, and refer to it as \((t_2, q_2)\).

**Step 4.** Determine the coordinate of the middle point on the smooth curve that corresponds to \((t_1, q_1)\) with the value of \( q_1 \), as obtained from the following expression:

\[ q_1 = \sqrt{q_i q_2} \]  

(16-17)

The corresponding value of \( t_1 \) is read from the smooth curve at \( q_1 \).

\[ t_a = \frac{q_i - q_a}{q_a D_i} \]  

(16-16)
Step 5. Solve the following equation iteratively for \( b \):

\[
f(b) = t_2 \left( \frac{q_i}{q_1} \right)^b - t_1 \left( \frac{q_i}{q_2} \right)^b - (t_2 - t_1) = 0 \quad (16-18)
\]

The Newton-Raphson iterative method can be employed to solve the previous nonlinear function by using the following recursion technique:

\[
b^{k+1} = b^k - \frac{f(b^k)}{f'(b^k)} \quad (16-19)
\]

where the derivative, \( f'(b^k) \), is given by

\[
f'(b^k) = t_2 \left( \frac{q_i}{q_1} \right)^{b^k} \ln \left( \frac{q_i}{q_1} \right) - t_1 \left( \frac{q_i}{q_2} \right)^{b^k} \ln \left( \frac{q_i}{q_2} \right) \quad (16-20)
\]

Starting with an initial value of \( b = 0.5 \), that is, \( b^k = 0.5 \), the method will usually converge after 4–5 iterations when the convergence criterion is set at \( [b^{k+1} - b^k] \leq 10^{-6} \).

Step 6. Solve for \( D_i \) with Equation 16-4, by using the calculated value of \( b \) from Step 5 and the coordinate of a point on the smooth graph, for example, \( (t_2, q_2) \), to give

\[
D_i = \left( \frac{q_i}{q_2} \right)^b - 1 \quad \frac{b t_2}{b t_2} \quad (16-21)
\]

The next example illustrates the proposed methodology for determining \( b \) and \( D_i \).
Example 16-3

The following production data were reported by Ikoku (1984) for a gas well:

<table>
<thead>
<tr>
<th>Date</th>
<th>Time, years</th>
<th>( q_t ), MMscf/day</th>
<th>( G_p(t) ), MMscf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 1, 1979</td>
<td>0.0</td>
<td>10.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Jul 1, 1979</td>
<td>0.5</td>
<td>8.40</td>
<td>1.67</td>
</tr>
<tr>
<td>Jan 1, 1980</td>
<td>1.0</td>
<td>7.12</td>
<td>3.08</td>
</tr>
<tr>
<td>Jul 1, 1980</td>
<td>1.5</td>
<td>6.16</td>
<td>4.30</td>
</tr>
<tr>
<td>Jan 1, 1981</td>
<td>2.0</td>
<td>5.36</td>
<td>5.35</td>
</tr>
<tr>
<td>Jul 1, 1981</td>
<td>2.5</td>
<td>4.72</td>
<td>6.27</td>
</tr>
<tr>
<td>Jan 1, 1982</td>
<td>3.0</td>
<td>4.18</td>
<td>7.08</td>
</tr>
<tr>
<td>Jul 1, 1982</td>
<td>3.5</td>
<td>3.72</td>
<td>7.78</td>
</tr>
<tr>
<td>Jan 1, 1983</td>
<td>4.0</td>
<td>3.36</td>
<td>8.44</td>
</tr>
</tbody>
</table>

Estimate the future production performance for the next 16 years.

Solution

Step 1. Determine the type of decline that adequately represents the historical data. This can be done by constructing the following two plots:

- Plot \( q_t \) versus \( t \) on a semi log scale, as shown in Figure 16-5. The plot does not yield a straight line, and, thus, the decline is not exponential.
- Plot \( q_t \) versus \( G_p(t) \) on a semi log scale, as shown in Figure 16-6. The plot again does not produce a straight line, and, therefore, the decline is not harmonic.

The two generated plots indicate that the decline must be hyperbolic.

Step 2. From Figure 16-5, determine the initial flow rate, \( q_i \), by extending the smooth curve to intercept with the \( y \)-axis, at \( t = 0 \), to give

\[ q_i = 10 \text{ MMscf/day} \]

Step 3. Select the coordinate of the other end-point on the smooth curve as \( (t_2, q_2) \), to give

\[ t_2 = 4 \text{ years and } q_2 = 3.36 \text{ MMscf/day} \]
Figure 16-5. Rate–time plot for Example 16-3.

Figure 16-6. Rate-cumulative plot for Example 16-3.
Step 4. Calculate $q_1$ from Equation 16-17 and determine the corresponding time:

$$q_1 = \sqrt{q_i q_2} = \sqrt{(10)(3.36)} = 5.8 \text{MMscf/day}$$

the corresponding time $t_1 = 1.719$ years

Step 5. Given $b = 0.5$, solve Equation 16-18 iteratively for $b$:

$$f(b) = t_2 \left( \frac{q_i}{q_1} \right)^b - t_1 \left( \frac{q_i}{q_2} \right)^b - (t_2 - t_1)$$

$$f(b) = 4(1.725)^b - 1.719(2.976)^b - 2.26$$

and

$$f'(b^k) = t_2 \left( \frac{q_i}{q_1} \right)^{b^k} \ln \left( \frac{q_i}{q_1} \right) - t_1 \left( \frac{q_i}{q_2} \right)^{b^k} \ln \left( \frac{q_i}{q_2} \right)$$

$$f'(b^k) = 2.18 (1.725)^b - 1.875 (2.976)^b$$

with

$$b^{k+1} = b^k - \frac{f(b^k)}{f'(b^k)}$$

It is convenient to perform the iterative method by constructing the following table:

<table>
<thead>
<tr>
<th>K</th>
<th>$b^k$</th>
<th>$f(b^k)$</th>
<th>$f'(b^k)$</th>
<th>$b^{k+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.500000</td>
<td>7.57 (10^{-3})</td>
<td>-0.36850</td>
<td>0.520540</td>
</tr>
<tr>
<td>1</td>
<td>0.520540</td>
<td>-4.19 (10^{-4})</td>
<td>-0.40950</td>
<td>0.519517</td>
</tr>
<tr>
<td>2</td>
<td>0.519517</td>
<td>-1.05 (10^{-6})</td>
<td>-0.40746</td>
<td>0.519514</td>
</tr>
<tr>
<td>3</td>
<td>0.519514</td>
<td>-6.87 \times 10^{-9}</td>
<td>-0.40745</td>
<td>0.519514</td>
</tr>
</tbody>
</table>

The method converges after 3 iterations with a value of $b = 0.5195$.  

Reservoir Engineering Handbook

$$D_i = \frac{(q_i/q_2)^b - 1}{b t_2} = \frac{(10/3.36)^{0.5195} - 1}{(0.5195)(4)} = 0.3668 \text{ year}^{-1}$$

or, on a monthly basis,

$$D_{im} = \frac{0.3668}{12} = 0.0306 \text{ month}^{-1}$$

or, on a daily basis,

$$D_{id} = \frac{0.3668}{365} = 0.001 \text{ day}^{-1}$$

Step 7. Use Equations 16-4 and 16-8 to predict the future production performance of the gas well. Notice that in Equation 16-4 the denominator contains $D_i t$ and, therefore, the product must be dimensionless, or

$$q_t = \frac{10(10^6)}{\left[1 + 0.5195 D_i t\right]^{1/0.5195}} = \frac{(10)(10^6)}{\left[1 + 0.5195 (0.3668)(t)\right]^{1/0.5195}}$$

where $q_t =$ flow rate, MMscf/day

$t =$ time, years

$D_i =$ decline rate, year$^{-1}$

In Equation 16-8, the time basis in $q_i$ is expressed in days and, therefore, $D_i$ must be expressed in day$^{-1}$, or

$$G_p(t) = \left[ \frac{q_i}{D_i (1-b)} \right] \left[ 1 - \left( \frac{q_i}{q_i} \right)^{1-b} \right]$$

$$G_p(t) = \left[ \frac{(10)(10^6)}{(0.001)(1-0.5195)} \right] \left[ 1 - \left( \frac{q_i}{(10)(10^6)} \right)^{1-0.5195} \right]$$

The results of Step 7 are tabulated below and shown graphically in Figure 16-7.
### Figure 16-7. Decline-curve data for Example 3-18.

<table>
<thead>
<tr>
<th>Time, years</th>
<th>Actual q, MMscf/day</th>
<th>Calculated q, MMscf/day</th>
<th>Actual cumulative gas, MMscf</th>
<th>Calculated cumulative gas, MMscf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>8.4</td>
<td>8.392971</td>
<td>1.67</td>
<td>1.671857</td>
</tr>
<tr>
<td>1</td>
<td>7.12</td>
<td>7.147962</td>
<td>3.08</td>
<td>3.08535</td>
</tr>
<tr>
<td>1.5</td>
<td>6.16</td>
<td>6.163401</td>
<td>4.3</td>
<td>4.296641</td>
</tr>
<tr>
<td>2</td>
<td>5.36</td>
<td>5.37108</td>
<td>5.35</td>
<td>5.346644</td>
</tr>
<tr>
<td>2.5</td>
<td>4.72</td>
<td>4.723797</td>
<td>6.27</td>
<td>6.265881</td>
</tr>
<tr>
<td>3</td>
<td>4.18</td>
<td>4.188031</td>
<td>7.08</td>
<td>7.077596</td>
</tr>
<tr>
<td>3.5</td>
<td>3.72</td>
<td>3.739441</td>
<td>7.78</td>
<td>7.799804</td>
</tr>
<tr>
<td>4</td>
<td>3.36</td>
<td>3.36</td>
<td>8.44</td>
<td>8.44669</td>
</tr>
<tr>
<td>5</td>
<td>2.757413</td>
<td></td>
<td></td>
<td>9.557617</td>
</tr>
<tr>
<td>6</td>
<td>2.304959</td>
<td></td>
<td></td>
<td>10.477755</td>
</tr>
<tr>
<td>7</td>
<td>1.956406</td>
<td></td>
<td></td>
<td>11.252814</td>
</tr>
<tr>
<td>8</td>
<td>1.68208</td>
<td></td>
<td></td>
<td>11.914924</td>
</tr>
<tr>
<td>9</td>
<td>1.462215</td>
<td></td>
<td></td>
<td>12.487334</td>
</tr>
<tr>
<td>10</td>
<td>1.283229</td>
<td></td>
<td></td>
<td>12.987298</td>
</tr>
<tr>
<td>11</td>
<td>1.135536</td>
<td></td>
<td></td>
<td>13.427888</td>
</tr>
<tr>
<td>12</td>
<td>1.012209</td>
<td></td>
<td></td>
<td>13.819197</td>
</tr>
<tr>
<td>13</td>
<td>0.908144</td>
<td></td>
<td></td>
<td>14.169139</td>
</tr>
<tr>
<td>14</td>
<td>0.819508</td>
<td></td>
<td></td>
<td>14.484015</td>
</tr>
<tr>
<td>15</td>
<td>0.743381</td>
<td></td>
<td></td>
<td>14.768899</td>
</tr>
<tr>
<td>16</td>
<td>0.677503</td>
<td></td>
<td></td>
<td>15.027928</td>
</tr>
<tr>
<td>17</td>
<td>0.620105</td>
<td></td>
<td></td>
<td>15.264506</td>
</tr>
<tr>
<td>18</td>
<td>0.569783</td>
<td></td>
<td></td>
<td>15.481464</td>
</tr>
<tr>
<td>19</td>
<td>0.525414</td>
<td></td>
<td></td>
<td>15.681171</td>
</tr>
<tr>
<td>20</td>
<td>0.486091</td>
<td></td>
<td></td>
<td>15.86563</td>
</tr>
</tbody>
</table>
Gentry (1972) developed a graphical method for the coefficients $b$ and $D_i$, as shown in Figures 16-8 and 16-9. Arps’ decline-curve exponent, $b$, is expressed in Figure 16-8 in terms of the ratios $q_i/q$ and $G_p/(t q_i)$, with an upper limit for $q_i/q$ of 100. To determine the exponent $b$, enter the graph with the abscissa with a value of $G_p/(t q_i)$ that corresponds to the last data point on the decline curve and enter the coordinate with the value of the ratio of initial production rate to last production rate on the decline curve, $q_i/q$. The exponent $b$ is read by the intersection of these two values. The initial decline rate, $D_i$, can be determined from Figure 16-9 by entering the coordinate with the value of $q_i/q$ and moving to the right to the curve that corresponds to the value of $b$. The initial decline rate, $D_i$, can be obtained by reading the value on the abscissa divided by the time $t$ from $q_i$ to $q$.

![Figure 16-8. Relationship between production rate and cumulative production. (After Gentry, 1972.)](image-url)
Example 16-4

Using the data given in Example 8-18, recalculate the coefficients $b$ and $D_t$ by using Gentry’s graphs.

Solution

Step 1. Calculate the ratios $q_i/q$ and $G_p/(t q_i)$:

\[
q_i/q = 10/3.36 = 2.98 \\
G_p/(t q_i) = 8440/[(4 \times 365)(10)] = 0.58
\]

Step 2. Enter Figure 16-9 with the values of 2.98 and 0.5 to give

\[D_t \cdot t = 1.5\]

Solving for $D_t$ gives

\[D_t = 1.5/4 = 0.38 \text{ year}^{-1}\]
In many cases gas wells are not produced at their full capacity during their early life for various reasons, such as limited capacity of flow lines, transportation, low demands, or other types of restrictions. Figure 16-10 illustrates a model for estimating the time pattern of production where the rate is restricted.

Figure 16-10 shows that the well produces at a restricted flow rate of $q_r$ for a total time of $t_r$ with a cumulative production of $G_{pr}$. The proposed methodology of estimating the restricted time, $t_r$, is to set the total cumulative production, $G_{p(tr)}$, that would have occurred under normal decline from the initial well capacity, $q_i$, down to $q_r$ equal to $G_{pr}$. Eventually, the well will reach the time $t_r$ where it begins to decline with a behavior similar to that of other wells in the area. The proposed method for predicting the decline-rate behavior for a well under restricted flow is based on the assumption that the following data are available and applicable to the well:

- Coefficients of Arps’ equation, that is, $D_i$ and $b$, by analogy with other wells
- Abandonment (economic) gas flow rate, $q_a$
- Ultimate recoverable reserves, $G_{pa}$
- Allowable (restricted) flow rate, $q_r$
The methodology is summarized in the following steps:

**Step 1.** Calculate the initial well flow capacity, $q_i$, that would have occurred with no restrictions, as follows:

- For Exponential: $q_i = G_{pa} D_i + q_a$  \hfill (16-22)
- For Harmonic: $q_i = q_r \left[ 1 + \frac{D_i G_{pa}}{q_r} - \ln \left( \frac{q_r}{q_a} \right) \right]$  \hfill (16-23)
- For Hyperbolic:
  \[
  q_i = \left[ (q_r)^b + \frac{D_i b G_{pa}}{(q_r)^{1-b}} - \frac{b (q_r)^b}{1-b} \left[ 1 - \left( \frac{q_a}{q_r} \right)^{1-b} \right] \right]^{1/b} \hfill (16-24)
  \]

**Step 2.** Calculate the cumulative gas production during the restricted-flow-rate period:

- For Exponential: $G_{pr} = \frac{q_i - q_r}{D_i}$  \hfill (16-25)
- For Harmonic: $G_{pr} = \left( \frac{q_i}{D_i} \right) \ln \left( \frac{q_i}{q_r} \right)$  \hfill (16-26)
- For Hyperbolic: $G_{pr} = \left[ \frac{q_i}{D_i (1-b)} \right] \left[ 1 - \left( \frac{q_r}{q_i} \right)^{1-b} \right]$  \hfill (16-27)

**Step 3.** Regardless of the type of decline, calculate the total time of the restricted flow rate from

\[
  t_r = \frac{G_{pr}}{q_r} \hfill (16-28)
\]

**Step 4.** Generate the well-production performance as a function of time by applying the appropriate decline relationships, as given by Equations 16-3 through 16-14.
Example 16-5

The volumetric calculations on a gas well show that the ultimate recoverable reserves, $G_{pa}$, are 25 MMMscf of gas. By analogy with other wells in the area, the following data are assigned to the well.

- Exponential decline
- Allowable (restricted) production rate $q_r = 425$ MMscf/month
- Economic limit $q_a = 30$ MMscf/month
- Nominal decline rate = 0.044 month$^{-1}$

Calculate the yearly production performance of the well.

Solution

Step 1. Estimate the initial flow rate, $q_i$, from Equation 16-22:

$$q_i = G_{pa} D_i + q_a = (0.044)(25,000) + 30 = 1,130$$ MMscf/month

Step 2. Calculate the cumulative gas production during the restricted flow period by using Equation 16-25.

$$G_{pr} = \frac{q_i - q_r}{D_i} = \frac{1130 - 425}{0.044} = 16.023$$ MMscf

Step 3. Calculate the total time of the restricted flow from Equation 16-28:

$$t_r = \frac{G_{pr}}{q_r} = \frac{16,023}{425} = 37.7$$ months = 3.14 years

Step 4. The yearly production during the first 3 years is

$$q = (425)(12) = 5100$$ MMscf/year

The fourth year is divided into 1.68 months, that is, 0.14 years (of constant production) plus 10.32 months of declining production; therefore, cumulative gas production during the first 1.68 months:

$$G_{P\text{ during 1.68 months}} = (1.68)(425) = 714$$ MMscf

At the end of the fourth year:

$$q_t = q_i \exp(-D_i t) = 425 \exp[-0.044(10.32)] = 270$$ MMscf/month
and cumulative gas production for the last 10.32 months:

$$G_{p(t)} = \frac{(q_i - q_i)}{D_i} = \frac{425 - 270}{0.044} = 3523 \text{ MMscf}$$

Total production for the fourth year $= 714 + 3523 = 4237 \text{ MMscf}$

<table>
<thead>
<tr>
<th>Year</th>
<th>Production, MMscf/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5100</td>
</tr>
<tr>
<td>2</td>
<td>5100</td>
</tr>
<tr>
<td>3</td>
<td>5100</td>
</tr>
<tr>
<td>4</td>
<td>4237</td>
</tr>
</tbody>
</table>

The flow rate at the end of the fourth year, 270 MMscf/month, is set equal to the *initial flow rate at the beginning of the fifth year*. The flow rate at the end of the fifth year, $q_{end}$, is calculated from Equation 16-25 as

$$q_{end} = q_i \exp[-D_i (12)] = 270 \exp[-0.044 (12)] = 159 \text{ MMscf/month}$$

with a cumulative gas production of

$$G_p = \frac{q_i - q_{end}}{D_i} = \frac{270 - 159}{0.044} = 2523 \text{ MMscf}$$

For the sixth year,

$$q_{end} = 159 \exp[-0.044 (12)] = 94 \text{ MMscf/month}$$

$$G_p = \frac{159 - 94}{0.044} = 1482 \text{ MMscf}$$
Results of this procedure are then tabulated:

<table>
<thead>
<tr>
<th>t, years</th>
<th>$Q_t$, MMscf/month</th>
<th>$Q_{end}$, MMscf/month</th>
<th>Yearly Production, MMscf/year</th>
<th>Cumulative Production, MMMscf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>425</td>
<td>425</td>
<td>5100</td>
<td>5.100</td>
</tr>
<tr>
<td>2</td>
<td>425</td>
<td>425</td>
<td>5100</td>
<td>10.200</td>
</tr>
<tr>
<td>3</td>
<td>425</td>
<td>425</td>
<td>5100</td>
<td>15.300</td>
</tr>
<tr>
<td>4</td>
<td>425</td>
<td>270</td>
<td>4237</td>
<td>19.537</td>
</tr>
<tr>
<td>5</td>
<td>270</td>
<td>159</td>
<td>2523</td>
<td>22.060</td>
</tr>
<tr>
<td>6</td>
<td>159</td>
<td>94</td>
<td>1482</td>
<td>23.542</td>
</tr>
<tr>
<td>7</td>
<td>94</td>
<td>55</td>
<td>886</td>
<td>24.428</td>
</tr>
<tr>
<td>8</td>
<td>55</td>
<td>33</td>
<td>500</td>
<td>24.928</td>
</tr>
</tbody>
</table>

Reinitialization of Data

Fetkovich (1971) points out that there are several obvious situations where rate–time data must be reinitialized for reasons that include among others,

- The drive or production mechanism has changed
- An abrupt change in the number of wells on a lease or a field due to infill drilling
- Changing the size of tubing would change $q_i$ and also the decline exponent, $b$

Provision of a well is not limited by tubing or equipment; the effects of stimulation will result in a change in deliverability, $q_i$, and possibly the remaining recoverable gas. However, the decline exponent, $b$, normally can be assumed constant. Fetkovich et al. (1996) suggested a rule-of-thumb equation to approximate an increase in rate due to stimulation:

$$(q_t)_{new} = \left[ \frac{7 + s_{old}}{7 + s_{new}} \right] (q_t)_{old}$$

where $(q_t)_{old}$ = producing rate just prior to stimulation

$s$ = skin factor

Arps’ equation (Equation 16-1) can be expressed as

$$q_t = \frac{(q_t)_{new}}{(1 + b t (D_t)_{new})^{1/b}}$$
with

$$(D_i)_{new} = \frac{(q_i)_{new}}{(1 - b)G}$$

where $G = \text{gas-in-place, scf}$

**TYPE-CURVE ANALYSIS**

The type-curve analysis approach was introduced to the petroleum industry by Agarwal et al. (1970) as a valuable tool when used in conjunction with conventional semi log plots. A type curve is a graphical representation of the theoretical solutions to flow equations. Type-curve analysis consists of finding the theoretical type curve that “matches” the actual response from a test well and the reservoir when subjected to changes in production rates or pressures. The match can be found graphically by physical superposition of a graph of actual test data on a similar graph of type curve(s) and searching for the type curve that provides the best match. Since type curves are plots of theoretical solutions to transient and pseudosteady-state flow equations, they are usually presented in terms of dimensionless variables, for example,

- dimensionless pressure, $p_D$
- dimensionless time, $t_D$
- dimensionless radius, $r_D$, and
- dimensionless wellbore storage, $C_D$

rather than real variables (e.g., $\Delta p$, $t$, $r$, and $C$). The reservoir and well parameters, such as permeability and skin, can then be calculated from the dimensionless parameters defining that type curve.

Any variable can be made “dimensionless” when multiplied by a group of constants with opposite dimensions, but the choice of this group will depend on the type of problem to be solved. For example, to create the dimensionless pressure drop, $p_D$, the actual pressure drop $\Delta p$ in psi is multiplied by group $A$ with units of psi$^{-1}$, or

$$p_\Delta = A\Delta p$$

Finding a group $A$ that makes a variable dimensionless is derived from equations that describe reservoir fluid flow. To introduce this concept,
recall Darcy’s equation (Chapter 6), which describes the radial, incompressible, steady-state flow as expressed by

$$Q = \frac{k h}{141.2 B \mu [\ln(r_e / r_{wa}) - 0.5]} \Delta p$$

(16-29)

where $r_{wa}$ is the apparent (effective) wellbore radius, as defined by Equation 6-152 in terms of the skin factors by

$$r_{wa} = r_w e^{-s}$$

Group A can be then defined by rearranging Darcy’s equation as:

$$\ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} = \left[ \frac{k h}{141.2 Q B \mu} \right] \Delta p$$

Because the left-hand side of the previous equation is dimensionless, the right-hand side must be accordingly dimensionless. This suggests that the term $((k h/(141.2 Q B \mu))$ is essentially a group A with units of psi$^{-1}$ that defines the dimensionless variable $p_D$, or

$$p_D = \left[ \frac{k h}{141.2 Q B \mu} \right] \Delta p$$

(16-30)

Taking the logarithm of both sides of the above equation gives

$$\log(p_D) = \log(\Delta p) + \log\left( \frac{k h}{141.2 Q B \mu} \right)$$

(16-31)

where $Q =$ flow rate, STB/day

$B =$ formation, volume factor, bbl/STB

$\mu =$ viscosity, cp

For a constant flow rate, Equation 16-31 indicates that the logarithm of dimensionless pressure drop, $\log(p_D)$, will differ from the logarithm of actual pressure drop, $\log(\Delta p)$, by a constant amount:

$$\log\left( \frac{k h}{141.2 Q B \mu} \right)$$
Similarly, the dimensionless time, $t_D$, is given in Chapter 6 by Equation 6-87 as

$$t_D = \left[ \frac{0.0002637k}{\phi \mu c_t r_w^2} \right] t$$

Taking the logarithm of both sides of the above equation gives

$$\log(t_D) = \log(t) + \log\left[ \frac{0.0002637k}{\phi \mu c_t r_w^2} \right]$$  \hspace{1cm} (16-32)

where $t = \text{time, hours}$

$c_t = \text{total compressibility coefficient, psi}^{-1}$

$\phi = \text{porosity}$

Hence, a graph of $\log(\Delta p)$ versus $\log(t)$ will have an identical shape (i.e., parallel) to a graph of $\log(p_D)$ versus $\log(t_D)$, although the curve will be shifted by $\log[kh/(141.2QB\mu)]$ vertically in pressure and $\log[0.0002637k/(\phi \mu c_t r_w^2)]$ horizontally in time. This concept is illustrated in Figure 16-11.

![Figure 16-11. Concept of type curves.](image)
Not only do these two curves have the same shape, but if they are moved relative to each other until they coincide or “match,” the vertical and horizontal displacements required to achieve the match are related to these constants in Equations 16-31 and 16-32. Once these constants are determined from the vertical and horizontal displacements, it is possible to estimate reservoir properties such as permeability and porosity. This process of matching two curves through the vertical and horizontal displacements and determining the reservoir or well properties is called type-curve matching.

Consider the Ei-function solution to the diffusivity equations, as given in Chapter 6 by Equation 6-78:

\[ p(r, t) = p_i + \left[ \frac{70.6 \cdot Q \cdot B \cdot \mu}{k \cdot h} \right] \cdot \text{Ei} \left( \frac{-948 \cdot \phi \cdot \mu \cdot c \cdot r^2}{k \cdot t} \right) \]

This relationship can be expressed in a dimensionless form by manipulation of the expression, to give

\[ \frac{p_i - p(r, t)}{\frac{141.2 \cdot Q \cdot B \cdot \mu}{k \cdot h}} = -\frac{1}{2} \cdot \text{Ei} \left[ \frac{-(r/r_w)^2}{4 \left( \frac{0.0002637}{\phi \cdot \mu \cdot c \cdot r^2} \right) \cdot k \cdot t} \right] \]

From the definition of the dimensionless variables \( p_D, t_D, \) and \( r_D \), this relation can be expressed in terms of these dimensionless variables:

\[ p_D = -\frac{1}{2} \cdot \text{Ei} \left( -\frac{r_D^2}{4 \cdot t_D} \right) \quad (16-33) \]

It should be noted that when \( t_D/r_D^2 > 25 \), Equation 16-33 can be approximated by

\[ p_D = \frac{1}{2} \left[ \ln \frac{t_D}{r_D^2} + 0.080907 \right] \]

Notice that

\[ \frac{t_D}{r_D^2} = \left[ \frac{0.0002637 \cdot k}{\phi \cdot \mu \cdot c \cdot r^2} \right] \cdot t \]
Taking the logarithm of both sides of the above equation gives

$$\log \left( \frac{t_D}{r_D^2} \right) = \log \left[ \frac{0.0002637 k}{\phi \mu c_t r^2} \right] + \log(t) \quad (16-34)$$

Equations 16-31 and 16-34 indicate that a graph of \( \log(\Delta p) \) versus \( \log(t) \) will have an **identical shape** (i.e., will be parallel) to a graph of \( \log(p_D) \) versus \( \log(t_D/r_D^2) \), although the curve will be shifted by \( \log(kh141.2/QB\mu) \) vertically in pressure and \( \log(0.0002637k/\phi \mu c_t r^2) \) horizontally in time. When these two curves are moved relative to each other until they coincide or “match,” the vertical and horizontal movements, in mathematical terms, are given by

$$\left( \frac{p_D}{\Delta p} \right)_{MP} = \frac{k h}{141.2 QB\mu} \quad (16-35)$$

and

$$\left( \frac{t_D/r_D^2}{t} \right)_{MP} = \frac{0.0002637 k}{\phi \mu c_t r^2} \quad (16-36)$$

The subscript “MP” denotes a **match point**.

A more practical solution to the diffusivity equation, then, is a plot of the dimensionless \( p_D \) versus \( t_D/r_D^2 \), as shown in Figure 16-12, which can be used to determine the pressure at any time and distance from the producing well. Figure 16-12 is basically a type curve that is mostly used in interference tests when analyzing pressure-response data in a shut-in observation well at a distance \( r \) from an active producer or injector well.

In general, the type-curve approach employs the flowing procedure that will be illustrated by the use of Figure 16-12:

**Step 1.** Select the proper type curve (e.g., Figure 16-12).

**Step 2.** Place a tracing paper over Figure 16-12 and construct a log-log scale that has the same dimensions as those of the type curve. This can be achieved by tracing the major and minor grid lines from the type curve to the tracing paper.
Step 3. Plot the well-test data in terms of $\Delta p$ versus $t$ on the tracing paper.

Step 4. Overlay the tracing paper on the type curve and slide the actual data plot, keeping the x- and y-axes of both graphs parallel, until the actual data point curve coincides with or matches the type curve.

Step 5. Select any arbitrary match point (MP), such as an intersection of major grid lines, and record $(\Delta p)_\text{MP}$ and $(t)_\text{MP}$ from the actual data plot and the corresponding values of $(p_D)_\text{MP}$ and $(t_D/t_D^2)\text{MP}$ from the type curve.

Step 6. Using the match point, calculate the properties of the reservoir.

Example 16-5 illustrates the convenience of using the type-curve approach in an interference test for 48 hours followed by a falloff period for 100 hours.
Example 16-6

During an interference test, water was injected at 170 bbl/day for 48 hours in an injection well. The pressure response in an observation well 119 ft away from the injector is as follows:

<table>
<thead>
<tr>
<th>t, hours</th>
<th>p, psig</th>
<th>Δp_{ws} = p_i - p, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>p_i = 0</td>
<td>0</td>
</tr>
<tr>
<td>4.3</td>
<td>22</td>
<td>-22</td>
</tr>
<tr>
<td>21.6</td>
<td>82</td>
<td>-82</td>
</tr>
<tr>
<td>28.2</td>
<td>95</td>
<td>-95</td>
</tr>
<tr>
<td>45.0</td>
<td>119</td>
<td>-119</td>
</tr>
<tr>
<td>48.0</td>
<td>injection ends</td>
<td></td>
</tr>
<tr>
<td>51.0</td>
<td>109</td>
<td>-109</td>
</tr>
<tr>
<td>69.0</td>
<td>55</td>
<td>-55</td>
</tr>
<tr>
<td>73.0</td>
<td>47</td>
<td>-47</td>
</tr>
<tr>
<td>93.0</td>
<td>32</td>
<td>-32</td>
</tr>
<tr>
<td>142.0</td>
<td>16</td>
<td>-16</td>
</tr>
<tr>
<td>148.0</td>
<td>15</td>
<td>-15</td>
</tr>
</tbody>
</table>

Other given data include the following:

- Initial pressure, $p_i = 0$ psi
- Water FVF, $B_w = 1.00$ bbl/STB
- Total compressibility, $c_t = 9.0 \times 10^{-6}$ psi$^{-1}$
- Formation thickness, $h = 45$ ft
- Water viscosity, $\mu_w = 1.3$ cp
- Injection rate, $q = -170$ bbl/day

Calculate the reservoir permeability and porosity.

Solution

Step 1. Figure 16-13 shows a plot of the well-test data during the injection period (48 hours) in terms of $\Delta p$ versus $t$ on a tracing paper with the same scale dimensions as in Figure 16-12. Using the overlay technique with the vertical and horizontal movements, find the segment of the type curve that matches the actual data.

---

1 This example problem and the solution procedure were given by Earlougher, R., “Advanced Well Test Analysis,” SPE Monograph Series, SPE, Dallas, TX (1977).
Step 2. Select any point on the graph to be defined as a match point, as shown in Figure 16-13. Record \((\Delta p)_{MP}\) and \((t)_{MP}\) from the actual data plot and the corresponding values of \((p_D)_{MP}\) and \((t_D/r_D^2)_{MP}\) from the type curve, to give

- Type-curve match values:

  \[(p_D)_{MP} = 0.96, \quad (t_D/r_D^2)_{MP} = 0.94\]

- Actual data match values:

  \[\Delta p_{MP} = -100 \text{ psig}, \quad (t)_{MP} = 10 \text{ hours}\]

Step 3. Using Equations 16-35 and 16-36, solve for the permeability and porosity:

\[k = \frac{141.2QB\mu}{h} \left(\frac{p_D}{\Delta p}\right)_{MP} = \frac{141.2(-170)(1.0)(1.0)}{45} \left(\frac{0.96}{-100}\right)_{MP}\]

\[= 5.1 \text{ md}\]

**Figure 16-13.** Illustration of type curve matching for an interference test using the type curve. (After Earlougher, R., Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).
and:

\[
\phi = \frac{0.0002637 k}{\mu c_i r^2 [(t_D / r_D^2)/t]_{MP}} = \frac{0.0002637 (5.1)}{(1.0) (9.0 \times 10^{-6}) (119)^2 \times [0.94/10]_{MP}}
\]

\[
= 0.11
\]

To fully understand the power and convenience of using the dimensionless concept approach in solving engineering problems, consider the following example.

**Example 16-7**

An oil well is producing under transient (unsteady-state) flow conditions. The following properties are given:

- \(p_i = 3500 \text{ psi}\)
- \(B = 1.44 \text{ bbl/STB}\)
- \(c_i = 17.6 \times 10^{-6} \text{ psi}^{-1}\)
- \(\phi = 15\%\)
- \(\mu = 1.3 \text{ cp}\)
- \(h = 20 \text{ ft}\)
- \(Q = 360 \text{ STB/day}\)
- \(k = 22.9 \text{ md}\)
- \(s = 0\)

(a) Calculate the pressure at radii of 10 ft and 100 ft for the flowing times 0.1, 0.5, 1.0, 2.0, 5.0, 10, 20, 50, and 100 hours. Plot \([p_i - p(r,t)]\) versus \((t/r^2)\) on a log-log scale.

(b) Present the data from part a in terms of \([p_i - p(r,t)]\) versus \((t/r^2)\) on a log-log scale.

**Solution**

During transient flow, Equation 6-78 is designed to describe the pressure at any radius \(r\) and any time \(t\), as given by

\[
p(r, t) = p_i + \left[\frac{70.6 Q B \mu}{k h} \right] \text{Ei} \left[\frac{-948 \phi c_i r^2}{k t}\right]
\]

or

\[
p_i - p(r, t) = \left[\frac{-70.6 (360) (1.444) (1.3)}{(22.9) (20)} \right] \text{Ei} \left[\frac{-948 (0.15) (1.3) (17.6 \times 10^{-6}) r^2}{(22.9) t}\right]
\]
Values of “$p_i - p(r,t)$” are presented as a function of time and radius (i.e., at $r = 10$ feet and 100 feet) in the following table and graphically in Figure 16-14.

$$p_i - p(r,t) = -104 \text{Ei}\left[-0.0001418 \frac{r^2}{t}\right]$$

<table>
<thead>
<tr>
<th>$t$, hours</th>
<th>$r = 10$ feet</th>
<th>$r = 100$ feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t/r^2$</td>
<td>$p_i - p(r,t)$</td>
<td>$t/r^2$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.001</td>
<td>-1.51</td>
</tr>
<tr>
<td>0.5</td>
<td>0.005</td>
<td>-3.02</td>
</tr>
<tr>
<td>1.0</td>
<td>0.010</td>
<td>-3.69</td>
</tr>
<tr>
<td>2.0</td>
<td>0.020</td>
<td>-4.38</td>
</tr>
<tr>
<td>5.0</td>
<td>0.050</td>
<td>-5.29</td>
</tr>
<tr>
<td>10.0</td>
<td>0.100</td>
<td>-5.98</td>
</tr>
<tr>
<td>20.0</td>
<td>0.200</td>
<td>-6.67</td>
</tr>
<tr>
<td>50.0</td>
<td>0.500</td>
<td>-7.60</td>
</tr>
<tr>
<td>100.0</td>
<td>1.000</td>
<td>-8.29</td>
</tr>
</tbody>
</table>

**Figure 16-14.** Pressure profile at 10 feet and 100 feet as a function of time.
Figure 16-14 shows different curves for the two radii. Obviously, the same calculations can be repeated for any number of radii and, consequently, the same number of curves will be generated. However, the solution can be greatly simplified by examining Figure 16-15. This plot shows that when the pressure difference $p_i - p(r,t)$ is plotted versus $t/r^2$, the data for both radii form a common curve. In fact, the pressure difference for any reservoir radius will plot on this exact same curve.

For example, in the same reservoir, to calculate the pressure $p$ at 150 feet after 200 hours of transient flow:

$$t/r^2 = \frac{200}{150^2} = 0.0089$$

From Figure 16-15:

$$p_i - p(r,t) = 370 \text{ psi}$$

Thus,

$$p(r,t) = p_i - 370 = 5000 - 370 = 4630 \text{ psi}$$
Several investigators have employed the dimensionless-variables approach to determine reserves and to describe the recovery performance of hydrocarbon systems with time, notably the following:

- Fetkovich (1980)
- Carter (1985)
- Palacio and Blasingame (1993)
- Anash et al. (2000)
- Decline-curve analysis for fractured reservoirs

All the methods are based on defining a set of decline-curve dimensionless variables that includes:

- Decline-curve dimensionless rate, \( q_{Dd} \)
- Decline-curve dimensionless cumulative production, \( Q_{Dd} \)
- Decline-curve dimensionless time, \( t_{Dd} \)

The aforementioned methods were developed with the objective of providing the engineer with an additional convenient tool for estimating reserves and determining other reservoir properties for oil and gas wells using the available performance data. A review of these methods and their practical applications is given next.

1. **Fetkovich Type Curve**

Type-curve matching is an advanced form of decline analysis proposed by Fetkovich (1980). The author proposed that the concept of the dimensionless-variables approach can be extended for use in decline-curve analysis to simplify the calculations. He introduced the variables for decline-curve dimensionless flow rate, \( q_{Dd} \), and decline-curve dimensionless time, \( t_{Dd} \), that are used in all decline-curve and type-curve analysis techniques. Arps’ relationships can thus be expressed in the following dimensionless forms:

- **Hyperbolic:**
  \[
  \frac{q_{t}}{q_{t_0}} = \frac{1}{[1 + b D_{t} t_{D}]^{1/b}}
  \]

In a dimensionless form:

\[
q_{Dd} = \frac{1}{[1 + b t_{Dd}]^{1/b}} \quad (16-37)
\]
where the decline-curve dimensionless variables $q_{Dd}$ and $t_{Dd}$ are defined by

\[ q_{Dd} = \frac{q_i}{q_i} \quad (16-38) \]
\[ t_{Dd} = D_i t \quad (16-39) \]

- **Exponential:** \[ \frac{q_i}{q_i} = \frac{1}{\exp[D_i t]} \]

Similarly, \[ q_{Dd} = \frac{1}{\exp[t_{Dd}]} \quad (16-40) \]

- **Harmonic:** \[ \frac{q_i}{q_i} = \frac{1}{1 + D_i t} \]

or \[ q_{Dd} = \frac{1}{1 + t_{Dd}} \quad (16-41) \]

where $q_{Dd}$ and $t_{Dd}$ are the decline-curve dimensionless variables, as defined by Equations 16-38 and 16-39, respectively.

During the **boundary-dominated flow period**, that is, steady-state or semisteady-state flowing conditions, Darcy’s equation can be used to describe the initial flow rate, $q_i$:

\[ q_i = \frac{0.00708 k h \Delta p}{B \mu \ln \left( \frac{r_e}{r_{wa}} \right)} - \frac{1}{2} = \frac{k h (p_i - p_{wi})}{141.2 B \mu \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2}} \]

where $q =$ flow rate, STB/day

- $B =$ formation, volume factor, bbl/STB
- $\mu =$ viscosity, cp
- $k =$ permeability, md
- $h =$ thickness, ft
- $r_e =$ drainage radius, ft
- $r_{wa} =$ apparent (effective) wellbore radius, ft
The ratio \( r_e/r_{wa} \) is commonly referred to as the dimensionless drainage radius \( r_D \):

\[
r_D = r_e / r_{wa} \tag{16-42}
\]

with

\[
r_{wa} = r_w e^{-s}
\]

The ratio \( r_e/r_{wa} \) in Darcy’s equation can be replaced with \( r_D \) to give

\[
q_i = \frac{k h (p_i - p_{wt})}{141.2 B \mu \left[ \ln(r_D) - \frac{1}{2} \right]}
\]

Rearranging Darcy’s equation gives

\[
\left[ 141.2 B \mu \right] q_i = \frac{1}{k h \Delta p} \ln \left( r_D \right) - \frac{1}{2}
\]

It is obvious that the right-hand side of the previous equation is dimensionless, which indicates that the left-hand side of the equation is also dimensionless. This relationship thus defines the dimensionless rate \( q_{D} \) as follows:

\[
q_{D} = \frac{141.2 B \mu q_i}{k h \Delta p} = \frac{1}{\ln(r_D) - \frac{1}{2}} \tag{16-43}
\]

Recall the dimensionless form of the diffusivity equation from Chapter 6, Equation 6-90:

\[
\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}
\]

Fetkovich (1980) demonstrated that the analytical solutions to these equations, the transient-flow diffusivity equation and the pseudosteady-state decline-curve equation, could be combined and presented in a family of log-log dimensionless curves. To develop this link between the two flow regimes, Fetkovich expressed the decline-curve dimensionless variables
q_{Dd} and t_{Dd} in terms of the transient dimensionless rate q_D and time t_D. Combining Equation 16-38 with Equation 16-43 gives

\[ q_{Dd} = \frac{q_t}{q_i} = \frac{q_t}{k h (p_i - p)} \left( \frac{141.2 B \mu}{\ln (r_D) - \frac{1}{2}} \right) \]

or

\[ q_{Dd} = q_i \left[ \ln (r_D) - \frac{1}{2} \right] \]

Fetkovich expressed the decline-curve dimensionless time t_{Dd} in terms of the transient dimensionless time t_D in this way:

\[ t_{Dd} = \frac{t_D}{\frac{1}{2} \left[ r_D^2 - 1 \right]} \left[ \ln (r_D) - \frac{1}{2} \right] \] (16-44)

Replacing the dimensionless time t_D with Equation 6-87 gives

\[ t_{Dd} = \frac{1}{\frac{1}{2} \left[ r_D^2 - 1 \right]} \left[ \ln (r_D) - \frac{1}{2} \right] \left[ \frac{0.006328 t}{\phi (\mu c_i) r_{wa}^2} \right] \] (16-45)

Although Arps’ exponential and hyperbolic equations were developed empirically on the basis of production data, Fetkovich was able to give a physical basis to Arps’ coefficients. Equations 16-39 and 16-46 indicate that the initial decline rate, D_i, can be defined mathematically by the following expression:

\[ D_i = \frac{1}{\frac{1}{2} \left[ r_D^2 - 1 \right]} \left[ \ln (r_D) - \frac{1}{2} \right] \left[ \frac{0.006328}{\phi (\mu c_i) r_{wa}^2} \right] \] (16-46)

Fetkovich arrived at his unified type curve, as shown in Figure 16-16, by solving the dimensionless form of the diffusivity equation using the constant-terminal solution approach for several assumed values of r_D and
t_{Dd} and the solution to Equation 16-37 as a function of t_{Dd} for several values of b ranging from 0 to 1.

Notice for Figure 16-16 that all curves coincide and become indistinguishable at t_{Dt} ≈ 0.3. Any data existing before a t_{Dt} of 0.3 will appear to represent exponential decline regardless of the true value of b and, thus, will plot as a straight line on a semi log scale. With regard to the initial rate q_i, it is not the actual producing rate at early time; it is very specifically a pseudosteady-state rate at the surface. This pseudo-state rate can be substantially less than the actual early time transient flow rates that would be produced from low-permeability wells with large negative skins.

The basic steps used in Fetkovich type-curve matching of declining rate–time data are as follows:

Step 1. Plot the historical flow rate, q_i, versus time, t, in any convenient units on log-log paper or tracing paper with the same logarithmic cycles as in the Fetkovich type curve.

Step 2. Place the tracing-paper data plot over the type curve and slide the tracing paper with plotted data, keeping the coordinate axes parallel, until the actual data points match one of the type curves with a specific value of b.
Because decline type-curve analysis is based on boundary-dominated flow conditions, there is no basis for choosing the proper b values for future boundary-dominated production if only transient data are available. In addition, because of the similarity of curve shapes, unique type-curve matches are difficult to obtain with transient data only. If it is apparent that boundary-dominated (i.e., pseudosteady-state) data are present and can be matched on a curve for a particular value of b, the actual curve can simply be extrapolated following the trend of the type curve into the future.

**Step 3.** From the match of the particular type curve of Step 2, record values of the reservoir dimensionless radius \( r_e/r_{wa} \) and the parameter \( b \).

**Step 4.** Select any convenient match point on the actual data plot (\( q_t \) and \( t_{mp} \)) and the corresponding values lying beneath that point on the type-curve grid (\( q_{Dd}, t_{Dd} \)\( _{mp} \)).

**Step 5.** Calculate the initial surface gas flow rate, \( q_i \), at \( t = 0 \) from the rate match point:

\[
q_i = \left[ \frac{q_t}{q_{Dd}} \right]_{mp} \quad (16-47)
\]

**Step 6.** Calculate the initial decline rate, \( D_i \), from the time match point:

\[
D_i = \left[ \frac{t_{Dd}}{t} \right]_{mp} \quad (16-48)
\]

**Step 7.** Using the value of \( r_e/r_{wa} \) from Step 3 and the calculated value of \( q_i \), calculate the formation permeability, \( k \), by applying Darcy’s equation in one of the following three forms:

- Pseudo-pressure form:

\[
k = \frac{1422 T \left[ \ln \left( r_e/r_{wa} \right) - 0.5 \right] q_i}{h \left[ m(p_i) - m(p_wf) \right]} \quad (16-49)
\]
• Pressure-squared form:

\[ k = \frac{1422 T (\mu_g Z)_{avg} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - 0.5 \right] q_i}{h \left( p_i^2 - p_{wf}^2 \right)} \]  \hspace{1cm} (16-50)

• Pressure-approximation form:

\[ k = \frac{141.2 \times (10^3) T (\mu_g B_g) \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - 0.5 \right] q_i}{h \left( p_i - p_{wf} \right)} \]  \hspace{1cm} (16-51)

where:
- \( k \) = permeability, md
- \( p_i \) = initial pressure, psia
- \( p_{wf} \) = bottom-hole flowing pressure, psia
- \( m(p) \) = pseudo-pressure, psi²/cp
- \( q_i \) = initial gas flow rate, Mscf/day
- \( T \) = temperature, °R
- \( h \) = thickness, ft
- \( \mu_g \) = gas viscosity, cp
- \( Z \) = gas deviation factor
- \( B_g \) = gas formation volume factor, bbl/scf

**Step 8.** Determine the reservoir pore volume (PV) of the well drainage area at the beginning of the boundary-dominated flow from the following expression:

\[ PV = \frac{56.54 T}{(\mu_g c_i)_{avg} \left[ m(p_i) - m(p_{wf}) \right]} \left( \frac{q_i}{D_i} \right) \]  \hspace{1cm} (16-52)

or, in terms of pressure squared,

\[ PV = \frac{28.27 T (\mu_g Z)_{avg}}{(\mu_g c_i)_{avg} \left[ p_i^2 - p_{wf}^2 \right]} \left( \frac{q_i}{D_i} \right) \]  \hspace{1cm} (16-53)

with

\[ r_e = \sqrt{\frac{(PV)}{\pi h \phi}} \]  \hspace{1cm} (16-54)
Step 9. Calculate the skin factor, \( s \), from the \( r_e/r_{wa} \) matching parameter and the calculated values of \( A \) and \( r_e \) from Step 8.

\[
s = \ln \left( \frac{r_e}{r_{wa}} \right) \ln \left( \frac{r_w}{r_e} \right)
\]

(16-56)

Step 10. Calculate the initial gas-in-place, \( G \), from

\[
G = \frac{(PV)[1 - S_w]}{5.615B_{gi}}
\]

(16-57)

The initial gas-in-place can also be estimated from the following relationship:

\[
G = \frac{q_i}{D_i (1 - b)}
\]

(16-58)

where \( G \) = initial gas-in-place, scf

\( S_w \) = initial water saturation

\( B_{gi} \) = gas formation volume factor at \( P_i \), bbl/scf

\( PV \) = pore volume, ft\(^3\)

An inherent problem when applying decline-curve analysis is having sufficient rate–time data to determine a unique value for \( b \) as shown in the Fetkovich type curve. It illustrates that the shorter the producing time, the
more the b value curves approach one another, which leads to the difficulty of obtaining a unique match. Arguably, applying the type-curve approach with only three years of production history may not be possible for some pools. Unfortunately, since time is plotted on a log scale, the production history becomes compressed so that even when incremental history is added, it may still be difficult to differentiate and clearly identify the appropriate decline exponent b.

The following example illustrates the use of the type-curve approach to determine reserves and other reservoir properties.

**Example 16-8**

Well A is a low-permeability gas well located in West Virginia. It produces from the Onondaga chert, which has been hydraulically fractured with 50,000 gal of 3% gelled acid and 30,000 lb of sand. A conventional Horner analysis of pressure buildup data on the well indicated the following:

\[
\begin{align*}
 p_i &= 3268 \text{ psia} \\
 m(P_i) &= 794.8 \times 10^6 \text{ psi}^2/\text{cp} \\
 k &= 0.082 \text{ md} \\
 s &= -5.4 
\end{align*}
\]

Fetkovich et al. (1987) provided the following additional data on the gas well:

\[
\begin{align*}
 p_{wf} &= 500 \text{ psia} \\
 \mu_{gi} &= 0.0172 \text{ cp} \\
 T &= 620^\circ R \\
 \varphi &= 0.06 \\
 S_w &= 0.35 \\
 m(P_{wf}) &= 20.8 \times 10^6 \text{ psi}^2/\text{cp} \\
 c_i &= 177 \times 10^{-6} \text{ psi}^{-1} \\
 h &= 70 \text{ ft} \\
 B_{gi} &= 0.000853 \text{ bbl/scf} \\
 r_w &= 0.35 \text{ ft} 
\end{align*}
\]

The rate–time data from the past 8 years were plotted and matched to a \( r_e/r_{wa} \) stem of 20 and b of 0.5, as shown in Figure 16-17. The resulting match point has the following coordinates:

\[
\begin{align*}
 q_t &= 1000 \text{ Mscf/day} \\
 t &= 100 \text{ days} \\
 q_{Dd} &= 0.58 \\
 t_{Dd} &= 0.126
\end{align*}
\]
Using the given data, calculate

(a) Permeability, \( k \)
(b) Drainage area, \( A \)
(c) Skin factor, \( s \)
(d) Gas-in-place, \( G \)

**Solution**

**Step 1.** Using the match point, calculate \( q_i \) and \( D_i \) by applying Equations 16-48 and 16-49, respectively.

\[
q_i = \left[ \frac{q_i}{D_i} \right]_{MP}
\]

\[
q_i = \frac{1000}{0.58} = 1724.1 \text{ Mscf/day}
\]
and:

\[
D_i = \left[ \frac{t_{pd}}{t} \right]_{np}
\]

\[
D_i = \frac{0.126}{100} = 0.00126 \text{ day}^{-1}
\]

**Step 2.** Calculate the permeability, \( k \), from Equation 16-50:

\[
k = \frac{1422 T \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.5 \right] q_i}{h \left[ m(p_i) - m(p_{wf}) \right]}
\]

\[
k = \frac{1422(620) \left[ \ln(20) - 0.5 \right] (1724.1)}{(70) [794.8 - 20.8] (10^6)} = 0.07 \text{ md}
\]

**Step 3.** Calculate the reservoir pore volume of the well drainage area using Equation 16-53:

\[
PV = \frac{56.54 T}{(\mu_{ig_i}) \left[ m(p_i) - m(p_{wf}) \right]} \left( \frac{q_i}{D_i} \right)
\]

\[
PV = \frac{(56.54)(620)}{(0.0172)(177)(10^{-6}) [794.8 - 20.8] (10^6)} \frac{1724.1}{0.00126}
\]

\[= 20.36 \times 10^6 \text{ ft}^3
\]

**Step 4.** Calculate the drainage radius and area by applying Equations 16-55 and 16-56:

\[
r_e = \sqrt{\frac{(PV)}{\pi h \phi}}
\]

\[
r_e = \sqrt{\frac{(20.36) 10^6}{\pi(70)(0.06)}} = 1242 \text{ ft}
\]
and:

\[ A = \frac{\pi re^2}{43,560} \]

\[ A = \frac{\pi (1,242)^2}{43,560} = 111 \text{ acres} \]

**Step 5.** Determine the skin factor from Equation 16-57:

\[ s = \ln \left( \frac{r_e}{r_{wa}} \left( \frac{r_w}{r_e} \right) \right) \]

\[ s = \ln \left( 20 \left( \frac{0.35}{1242} \right) \right) = -5.18 \]

**Step 6.** Calculate the initial gas-in-place using Equation 16-58:

\[ G = \frac{(PV)[1 - S_w]}{5.615B_{gi}} \]

\[ G = \frac{(20.36) \times 10^6 \left[ 1 - 0.35 \right]}{(5.615) \times (0.000853)} = 2.763 \text{Bscf} \]

The initial gas \( G \) can also be estimated from Equation 16-59, to give

\[ G = \frac{q_i}{D_i (1 - b)} \]

\[ G = \frac{1.7241 \times 10^6}{0.00126 (1 - 0.5)} \approx 2.737 \text{Bscf} \]
Limits of Exponent b and Decline Analysis of Stratified No-Crossflow Reservoirs

Most reservoirs consist of several layers with varying reservoir properties. No-crossflow reservoirs are perhaps the most prevalent and important, so reservoir heterogeneity is of considerable significance in long-term prediction and reserve estimates. In layered reservoirs with crossflow, adjacent layers can simply be combined into a single equivalent layer that can be described as a homogeneous layer by averaging reservoir properties of the crossflowing layers. As shown later in this section, the decline-curve exponent, b, for a single homogeneous layer ranges between 0 and a maximum value of 0.5. For layered no-crossflow systems, values of b range between 0.5 and 1 and therefore can be used to identify the stratification. These separated layers might have the greatest potential for increasing current production and recoverable reserves.

Recall the back-pressure equation, Equation (16-5):

\[ q_g = C (p_r^2 - p_{wf}^2)^n \]

where \( n \) = back-pressure curve exponent
\( C \) = performance coefficient
\( p_r \) = reservoir pressure

Fetkovich (1996) suggested that the Arps decline exponent b and the decline rate can be expressed in terms of the exponent n as follows:

\[ b = \frac{1}{2n} \left( (2n - 1) - \left( \frac{p_{wf}}{p_i} \right)^2 \right) \]

\[ D_i = 2n \left( \frac{q_i}{G} \right) \]

Equation 16-60 indicates that as the reservoir pressure, \( p_r \), approaches \( p_{wf} \) with depletion, all the nonexponential decline (\( b \neq 0 \)) will shift toward exponential decline (\( b = 0 \)) as depletion proceeds. Equation 16-60 also suggests that if the well is producing at a very low bottom-hole flowing
pressure (i.e., \( p_{wf} = 0 \) or \( p_{wf} < p_i \)), the equation can be reduced to the following expression:

\[
b = 1 - \frac{1}{2n}
\]  

(16-61)

The exponent \( n \) from a gas well back-pressure performance curve can therefore be used to calculate or estimate \( b \) and \( D_i \). Equation 16-61 provides the physical limits of \( b \), which is between 0 and 0.5, over the accepted theoretical range of \( n \), which is between 0.5 and 1.0 for a single-layer homogeneous system, as shown in the following table:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(high ( k )) 0.50</td>
<td>0.0</td>
</tr>
<tr>
<td>0.56</td>
<td>0.1</td>
</tr>
<tr>
<td>0.62</td>
<td>0.2</td>
</tr>
<tr>
<td>0.71</td>
<td>0.3</td>
</tr>
<tr>
<td>0.83</td>
<td>0.4</td>
</tr>
<tr>
<td>(low ( k )) 1.00</td>
<td>0.5</td>
</tr>
</tbody>
</table>

However, the harmonic decline exponent, \( b = 1 \), cannot be obtained from the back-pressure exponent. The \( b \) value of 0.4 should be considered a good limiting value for gas wells when not clearly defined by actual production data.

The following is a tabulation of the values of \( b \) that should be expected for homogeneous single-layer or layered crossflow systems.

<table>
<thead>
<tr>
<th>( b )</th>
<th>System Characterization and Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>• Gas wells undergoing liquid loading</td>
</tr>
<tr>
<td></td>
<td>• Wells with high back pressure</td>
</tr>
<tr>
<td></td>
<td>• High-pressure gas</td>
</tr>
<tr>
<td></td>
<td>• Low-pressure gas with a back-Pressure curve exponent of ( n = 0.5 )</td>
</tr>
<tr>
<td></td>
<td>• Poor waterflood performance (oil wells)</td>
</tr>
<tr>
<td></td>
<td>• Gravity drainage with no solution gas (oil wells)</td>
</tr>
<tr>
<td></td>
<td>• Solution gas drive with unfavorable ( k_g/k_o ) (oil wells)</td>
</tr>
<tr>
<td>0.3</td>
<td>• Typical for solution-gas-drive reservoirs</td>
</tr>
<tr>
<td>0.4 – 0.5</td>
<td>• Typical for gas wells, ( b = 0.5 ) for ( p_{wf} = 0 ); ( b = 0.4 ) for ( p_{wf} = 0.1p_i )</td>
</tr>
<tr>
<td>0.5</td>
<td>• Gravity drainage WITH solution gas and for water-drive oil reservoirs</td>
</tr>
<tr>
<td>Undeterminable</td>
<td>• Constant-rate or increasing-rate production period</td>
</tr>
<tr>
<td></td>
<td>• Flow rates are all in transient or infinite-acting period</td>
</tr>
<tr>
<td>0.5 &lt; ( b ) &lt; 0.9</td>
<td>• Layered or composite reservoir</td>
</tr>
</tbody>
</table>
The significance of the b value is that for a single-layer reservoir, the value of b will lie between 0 and 0.5. With layered no-crossflow performance, however, the b value can be between 0.5 and 1.0. As pointed out by Fetkovich (1997), the further the b value is driven toward 1.0, the more unrecovered reserves remain in the tight low-permeability layer and the greater potential there is to increase production and recoverable reserves through stimulation of the low-permeability layer. This suggests that decline-curve analysis can be used to recognize and identify layered no-crossflow performance using only readily available historical production data. Recognition of the layers that are not being adequately drained compared to other layers, that is, differential depletion, is where the opportunity lies. Stimulation of the less-productive layers can allow an increase in both production and reserves. Figure 16-18 represents the standard Arps depletion/decline curves, as presented by Fetkovich (1997). Ten curves are shown, each described by a b value that ranges between 0 and 1 in increments of 0.1. All of the values have meaning and should be understood for the proper application of decline-curve analysis. When decline-curve analysis yields a b value greater than 0.5 (layered no-crossflow production), it is inaccurate to simply make a prediction from the match-point values. This is because the match point represents a best fit of the surface production data, which include production

![Figure 16-18. Depletion decline curves. (After Fetkovich, 1997, copyright SPE 1997.)](image-url)
data from all layers. Multiple combinations of layer production values can give the same composite curve and, therefore, unrealistic forecasts in late time may be generated.

To demonstrate the effect of the layered no-crossflow reservoir system on the exponent b, Fetkovich et al. (1990) evaluated the production-depletion performance of a two-layered gas reservoir producing from two noncommunicated layers. The field produces from 10 wells and contains an estimated 1.5 Bscf gas initially in place at an initial reservoir pressure of 428 psia. The reservoir has a gross thickness of 350 ft and a shale barrier with an average thickness of 50 ft that is clearly identified across the field and separates the two layers. Core data indicate a bimodal distribution with a permeability ratio between 10:1 and 20:1.

A type-curve analysis and regression fit of the total field composite log (q_i) versus log (t) yielded b = 0.89, which is identical to all values obtained from individual well analysis. To provide a quantitative analysis and an early recognition of a non-crossflow layered reservoir, Fetkovich (1980) expressed the rate–time equation for a gas well in terms of the back-pressure exponent, n, with a constant p_{wf} of 0. The derivation is based on a combination of Arps’ hyperbolic equation with the material balance equation (i.e., p/z versus G_p) and back-pressure equation to give the following:

**For 0.5 < n < 1, 0 < b < 0.5:**

\[
q_i = \frac{q_i}{\left[1 + (2n - 1)\left(\frac{q_i}{G}\right)t\right]\left[1 - \frac{2n - 1}{2n}\right]}^{\frac{1}{2n - 1}} \quad (16-62)
\]

\[
G_{p(t)} = G\left\{1 - \left[1 + (2n - 1)\left(\frac{q_i}{G}\right)t\right]\left[1 - \frac{1}{2n}\right]\right\} \quad (16-63)
\]

**For n = 0.5, b = 0:**

\[
q_i = q_i \exp\left[-\left(\frac{q_i}{G}\right)t\right] \quad (16-64)
\]
\[
G_{p(t)} = G \left[ 1 - \exp \left( -\left( \frac{q_t}{G} \right) t \right) \right] \tag{16-65}
\]

- For \( n = 1, b = 0.5 \):

\[
q_i = \frac{q_i}{1 + \left( \frac{q_i}{G} \right) t}^2 \tag{16-66}
\]

\[
G_{p(t)} = G - \frac{G}{1 + \left( \frac{q_i t}{G} \right)} \tag{16-67}
\]

These relationships are based on \( P_{wf} = 0 \), which implies that \( q_i = q_{imax} \), as given by

\[
q_i = q_{imax} \pm \frac{k h p_i^2}{1422 T (\mu_g Z)_{avg} \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} \tag{16-68}
\]

where

- \( q_{imax} \) = stabilized absolute open-flow potential, i.e., at \( P_{wf} = 0 \), Mcf/day
- \( G \) = initial gas-in-place, Mcf
- \( q_t \) = gas flow rate at time \( t \), Mcf/day
- \( t \) = time
- \( G_{p(t)} \) = cumulative gas production at time \( t \), Mcf

For a commingled well producing from two layers at a constant \( P_{wf} \), the total flow rate \( (q_t)_{total} \) is essentially the sum of the flow rates from all layers, or

\[
(q_t)_{total} = (q_t)_1 + (q_t)_2
\]

where the subscripts 1 and 2 represent the more permeable layer and less permeable layer, respectively. For a hyperbolic exponent of \( b = 0.5 \), Equation 16-67 can be substituted into the above expression to give
Equation 16-70 indicates that only if the value of \( b = 0.5 \) for each layer yield a composite rate–time value of \( b = 0.5 \).

Mattar and Anderson (2003) presented an excellent review of methods that are available for analyzing production data using traditional and modern type curves. Basically, modern type-curve analysis methods incorporate the flowing pressure data along with production rates, and they use the analytical solutions to calculate hydrocarbon in place. Two important features of modern decline analysis that improve upon the traditional techniques are as follows:

• **Normalization of rates using flowing pressure drop:** Plotting a normalized rate \( \frac{q}{\Delta p} \) enables the effects of back-pressure changes to be accommodated in the reservoir analysis.

• **Handling the change in gas compressibility with pressure:** Using pseudo-time as the time function, instead of real time, enables the gas material balance to be handled rigorously as the reservoir pressure declines with time.

### 2. Carter Type Curve

Fetkovich originally developed his type curves for gas and oil wells that are producing at constant pressures. Carter (1985) presented a new set of type curves developed exclusively for the analysis of gas rate data. Carter noted that the changes in fluid properties with pressure significantly affect reservoir performance predictions. Of utmost importance is the variation in the gas viscosity–compressibility product, \( \mu_g c_g \), which was ignored by Fetkovich. Carter developed another set of decline curves for boundary-dominated flow that uses a new correlating parameter, \( \lambda \), to represent the changes in \( \mu_g c_g \) during depletion. The \( \lambda \) parameter, called the “dimensionless drawdown correlating parameter,”
is designated to reflect the magnitude of pressure drawdown on $\mu_g c_g$ and defined as follows:

$$\lambda = \frac{(\mu_g c_g)_i}{(\mu_g c_g)_{avg}}$$

or, equivalently,

$$\lambda = \frac{(\mu_g c_g)_i}{2} \left[ \frac{m(p_i) - m(p_{wf})}{p_i - p_{wf}} \right] \left[ \frac{p_i}{Z_i} - \frac{p_{wf}}{Z_{wf}} \right]$$

(16-71)

where

- $c_g$ = gas compressibility coefficient, psi$^{-1}$
- $m(p)$ = real gas pseudo-pressure, psi$^2$/cp
- $p_{wf}$ = bottom-hole flowing pressure, psi
- $p_i$ = initial pressure, psi
- $\mu_g$ = gas viscosity, cp
- $Z$ = gas deviation factor

For $\lambda = 1$, it indicates a negligible drawdown effect and corresponds to the $b = 0$ on the Fetkovich exponential decline curve. Values of $\lambda$ range between 0.55 and 1.0. The type curves presented by Carter are based on four specially defined dimensionless parameters:

- Dimensionless time, $t_D$
- Dimensionless rate, $q_D$
- Dimensionless geometry parameter, $\eta$, which characterizes the dimensionless radius, $r_{cD}$, and flow geometry
- Dimensionless drawdown correlating parameter, $\lambda$

Carter used a finite-difference radial-gas model to generate the data used to construct the type curves shown in Figure 16-19.

The following steps summarize the type-curve matching procedure.
Step 1. Using Equation 16-71 or Equation 16-72, calculate the parameter \( \lambda \).

\[
\lambda = \frac{(\mu_g c_g)_i}{(\mu_g c_g)_{\text{avg}}}
\]

or

\[
\lambda = \frac{(\mu_g c_g)_i}{2} \left[ \frac{m(p_i) - m(p_{wf})}{P_i - P_{wf}} \right] \frac{Z_i}{Z_{wf}}
\]

Figure 16-19. Radial-linear gas reservoir type curves. (After Carter, SPEJ 1985, copyright SPE 1985.)
Step 2. Plot gas rate, \( q \), in Mscf/day or MMscf/day as a function of time (\( t \)) in days using the same log-log scale as the type curves. If actual rate values are erratic or fluctuate, it may be best to obtain averaged values of rate by determining the slope of straight lines drawn through adjacent points spaced at regular intervals on the plot of cumulative production, \( G_p \), versus time. That is, slope = \( \frac{dG_p}{dt} = q_g \). The resulting plot of \( q_g \) versus \( t \) should be made on tracing paper or on a transparency so that it can be laid over the type curves for matching.

Step 3. Match the rate data to a type curve corresponding to the computed value of \( \lambda \) in Step 1. If the computed value of \( \lambda \) is not one of the values for which a type curve is shown, the needed curve can be obtained by interpolation and graphical construction.

Step 4. From the match, values of \( (q_D)_{mp} \) and \( (t_D)_{mp} \) corresponding to specific values for \( (q)_{mp} \) and \( (t)_{mp} \) are recorded. A value for the dimensionless geometry parameter \( \eta \) is also obtained from the match. It is strongly emphasized that late-time data points (boundary-dominated, pseudosteady-state flow condition) are to be matched in preference to early-time data points (unsteady-state flow condition) because matching some early rate data often will be impossible.

Step 5. Estimate the gas that would be recoverable if the average reservoir pressure were reduced from its initial value to \( P_{wf} \) from the following expression:

\[
\Delta G = G_i - G_{pwf} = \frac{(q t)_{mp}}{(q_D t_D)_{mp}} \frac{\eta}{\lambda}
\]  

(16-72)

Step 6. Calculate the initial gas-in-place, \( G_i \), from

\[
G_i = \begin{bmatrix}
\frac{p_i}{Z_i} \\
\frac{p_i - p_{wf}}{Z_i} \\
\frac{Z_i}{Z_{wf}}
\end{bmatrix} \Delta G
\]

(16-73)
Step 7. Estimate the drainage area of the gas well from

$$A = \frac{B_{gi} G_i}{43,560 \phi h (1 - S_{wi})}$$  \hspace{1cm} (16-74)

where $B_{gi} =$ gas formation volume factor at $P_i$, ft$^3$/scf

$A =$ drainage area, acres

$h =$ thickness, ft

$\phi =$ porosity

$S_{wi} =$ initial water saturation

Example 16-9

The following production and reservoir data were used by Carter (1985) to illustrate the proposed calculation procedure.

<table>
<thead>
<tr>
<th>$p$, psia</th>
<th>$\mu_g$, cP</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0143</td>
<td>1.0000</td>
</tr>
<tr>
<td>601</td>
<td>0.0149</td>
<td>0.9641</td>
</tr>
<tr>
<td>1201</td>
<td>0.0157</td>
<td>0.9378</td>
</tr>
<tr>
<td>1801</td>
<td>0.0170</td>
<td>0.9231</td>
</tr>
<tr>
<td>2401</td>
<td>0.0188</td>
<td>0.9207</td>
</tr>
<tr>
<td>3001</td>
<td>0.0208</td>
<td>0.9298</td>
</tr>
<tr>
<td>3601</td>
<td>0.0230</td>
<td>0.9486</td>
</tr>
<tr>
<td>4201</td>
<td>0.0252</td>
<td>0.9747</td>
</tr>
<tr>
<td>4801</td>
<td>0.0275</td>
<td>1.0063</td>
</tr>
<tr>
<td>5401</td>
<td>0.0298</td>
<td>1.0418</td>
</tr>
</tbody>
</table>

$p_i = 5400$ psia

$T = 726^\circ$R

$\phi = 0.070$

$\lambda = 0.55$

$p_{wf} = 500$ psi

$h = 50$ ft

$S_{wi} = 0.50$

<table>
<thead>
<tr>
<th>Time, days</th>
<th>$q_t$, MMscf/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>8.300</td>
</tr>
<tr>
<td>10.20</td>
<td>3.400</td>
</tr>
<tr>
<td>20.50</td>
<td>2.630</td>
</tr>
<tr>
<td>40.90</td>
<td>2.090</td>
</tr>
</tbody>
</table>
Calculate the initial gas-in-place and the drainage area.

Solution

Step 1. The calculated value of $\lambda$ is given as 0.55 and, therefore, the type curve for a $\lambda$ value of 0.55 can be used directly from Figure 16-19.

Step 2. Plot the production data, as shown in Figure 16-20, on the same log-log scale as Figure 16-16 and determine the match points of the following:

\[
(q)_{mp} = 1.0 \text{ MMscf/day} \\
(t)_{mp} = 1000 \text{ days} \\
(qD)_{mp} = 0.605 \\
(tD)_{mp} = 1.1 \\
\eta = 1.045
\]

Step 3. Calculate $\Delta G$ from Equation 16-73.

\[
\Delta G = G_i - G_{pwl} = \frac{(q \ t)_{mp}}{(qD \ tD)_{mp}} \frac{\eta}{\lambda} \\
\Delta G = \frac{(1) (1000)}{(0.605) (1.1)} \frac{1.045}{0.55} = 2860 \text{ MMscf}
\]
Step 4. Estimate the initial gas-in-place by applying Equation 16-74:

\[
G_i = \left[ \frac{p_i}{Z_i} - \frac{p_{wf}}{Z_{wf}} \right] \Delta G
\]

\[
G_i = \left[ \frac{5400}{1.0418/500 - 1.0418/0.970} \right] \quad 2860 = 3176 \text{MMscf}
\]

Step 5. Calculate the gas formation volume factor, \( B_{gi} \), at \( p_i \):

\[
B_{gi} = 0.0287 \frac{Z_i T}{p_i} = 0.02827 \frac{1.0418 (726)}{5400} = 0.00396 \text{ft}^3/\text{scf}
\]
Step 6. Determine the drainage area from Equation 16-75:

\[ A = \frac{B_{gi} G_i}{43,560 \phi h (1 - s_{wi})} \]

\[ A = \frac{0.00396 \times (3176 \times 10^6)}{43,560 \times (0.070) \times (50) \times (1 - 0.50)} = 105 \text{ acres} \]

3. Palacio-Blasingame Type Curve

Palacio and Blasingame (1993) presented an innovative technique for converting gas well production data with variable rates and bottom-hole flowing pressures into “equivalent constant-rate liquid data” that allows the liquid solutions to be used to model gas flow. The reasoning for this approach is that the constant-rate type-curve solutions for liquid flow problems are well established from the traditional well-test analysis approach. The new solution for the gas problem is based on a material-balance-like time function and an algorithm that allows the following three things:

- The use of *decline curves that are specifically developed for liquids*
- Modeling of actual *variable rate–variable pressure* drop production conditions
- Explicit computation of *gas-in-place*

Under pseudosteady-state flow conditions, Equation 6-137 in Chapter 6 describes the radial flow of slightly compressible liquids:

\[ p_{wf} = \left[ p_i - \frac{0.23396 Q B_t}{A h \phi c_t} \right] - \frac{162.6 Q B \mu}{k h} \log \left[ \frac{4A}{1.781 C_A r_w^2} \right] \]

where

- \( k \) = permeability, md
- \( A \) = drainage area, ft\(^2\)
- \( C_A \) = shape factor
- \( Q \) = flow rate, STB/day
- \( t \) = time, hrs
- \( c_t \) = total compressibility coefficient, psi\(^{-1}\)
Expressing the time \( t \) in days and converting from log to a natural logarithm, \( \ln \), the above relation can be written as follows:

\[
\frac{p_i - p_{wf}}{q} = \frac{\Delta p}{q} = 70.6 \frac{B\mu}{kh} \ln \left[ \frac{4A}{1.781C_Ar_{wa}^2} \right] + \left[ \frac{5.615B}{A\phi c_t} \right] t \quad (16-75)
\]

or more conveniently as

\[
\frac{\Delta p}{q} = b_{pss} + m t \quad (16-76)
\]

The above expressions suggest that, under a pseudosteady-state flowing condition, a plot of \( \Delta p/q \) versus \( t \) on a Cartesian scale would yield a straight line with an intercept of \( b_{pss} \) and slope of \( m \):

**Intercept:**

\[
b_{pss} = 70.6 \frac{B\mu}{kh} \ln \left[ \frac{4A}{1.781C_Ar_{wa}^2} \right] \quad (16-77)
\]

**Slope:**

\[
m = \frac{5.615B}{A\phi c_t} \quad (16-78)
\]

where \( b_{pss} = \) constant in the pseudosteady-state (pss) equation

\[ t = \text{time, days} \]
\[ k = \text{permeability, md} \]
\[ A = \text{drainage area, ft}^2 \]
\[ q = \text{flow rate, STB/day} \]
\[ B = \text{formation volume factor, bbl/STB} \]
\[ C_A = \text{shape factor} \]
\[ c_t = \text{total compressibility, psi}^{-1} \]
\[ r_{wa} = \text{apparent (effective) wellbore radius, ft} \]

For a gas system flowing under pseudosteady-state conditions, Equation 6-139 in Chapter 6 describes the flow as follows:

\[
\frac{m(p_i) - m(p_{art})}{q} = \frac{\Delta m(p)}{q} = 711T \frac{4A}{kh} \left( \ln \frac{4A}{1.781C_Ar_{wa}^2} \right) + \left[ \frac{56.54T}{\phi(\mu_g c_g), A h} \right] t \quad (16-79)
\]
and in a linear form as

\[
\frac{\Delta m(p)}{q} = b_{\text{ps}} + mt
\]  

(16-80)

Similarly to the liquid system, Equation 16-81 indicates that a plot of \(\Delta m(p)/q\) versus \(t\) will form a straight line with the following features:

Intercept: \(b_{\text{ps}} = \frac{711 \text{ T}}{\text{k h}} \left( \ln \frac{4A}{1.781 C_A r_w^2} \right)\)

Slope: \(m = \frac{56.54 \text{ T}}{(\mu_g c_g)_i (\phi hA)} = \frac{56.54 \text{ T}}{(\mu_g c_g)_i \text{ (pore volume)}}\)

where \(q = \text{flow rate, Mscf/day}\)
\(A = \text{drainage area, ft}^2\)
\(T = \text{temperature, } ^\circ\text{R}\)
\(t = \text{flow time, days}\)

The linkage that allows for the conversion of gas-production data into equivalent constant-rate liquid data is based on the use of a new time function called pseudo-equivalent time or normalized material balance pseudo-time, defined as follows:

\[
t_a = \frac{(\mu_g c_g)_i}{q_t} \int \left[ \frac{q_t}{\mu_g c_g} \right] \text{d}t = \frac{(\mu_g c_g)_i}{q_t} \frac{Z_t G}{2p_i} \left[ m(\bar{p}_i) - m(\bar{p}) \right]
\]  

(16-81)

where
\(t_a = \text{pseudo-equivalent (normalized material balance) time, days}\)
\(t = \text{time, days}\)
\(G = \text{original gas-in-place, Mscf}\)
\(q_t = \text{gas flow rate at time } t, \text{ Mscf/day}\)
\(p = \text{average pressure, psi}\)
\(\mu_g = \text{gas viscosity at } \bar{p}, \text{ cp}\)
\(\bar{c}_g = \text{gas compressibility at } \bar{p}, \text{ psi}^{-1}\)
\(\bar{m}(p) = \text{normalized gas pseudo-pressure, psi}^2/\text{cp}\)
In order to perform decline-curve analysis under variable rates and pressures, the authors derived a theoretical expression for decline-curve analysis that combines the following elements:

- Material balance relation
- Pseudosteady-state equation
- Normalized material balance time function, \( t_a \)

to give the following relationship:

\[
\left[ \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right] b_{\text{pss}} = \frac{1}{1 + \left( \frac{m}{b_{\text{pss}}} \right) t_a}
\]  

(16-82)

where \( \bar{m}(p) \) is the normalized pseudo-pressure as defined by

\[
\bar{m}(p_i) = \frac{\mu_{gi} Z_i}{p_i} \left[ \frac{p}{\mu_g Z} \right] \int_0^p dp
\]  

(16-83)

\[
\bar{m}(p) = \frac{\mu_{gi} Z_i}{p_i} \left[ \frac{p}{\mu_g Z} \right] \int_0^p dp
\]  

(16-84)

and

\[
m = \frac{1}{G c_{ri}}
\]  

(16-85)

\[
b_{\text{pss}} = \frac{70.6 \mu_{gi} B_{gi}}{k_g h} \ln \left( \frac{4A}{1.781 C_A r_{wa}^2} \right)
\]  

(16-86)

where

- \( G \) = original gas-in-place, Mscf
- \( c_{gi} \) = gas compressibility at \( p_i \), psi\(^{-1}\)
- \( c_{ri} \) = total system compressibility at \( P_i \), psi\(^{-1}\)
- \( q_g \) = gas flow rate, Mscf/day
- \( k_g \) = effective permeability to gas, md
- \( \bar{m}(p) \) = normalized pseudo-pressure, psia
\( p_i = \text{initial pressure} \)
\( r_{wa} = \text{effective (apparent) wellbore radius, ft} \)
\( B_{gi} = \text{gas formation volume factor at } p_i, \text{ bbl/Mscf} \)

Notice that Equation 16-83 is essentially expressed in the same dimensionless form as the Fetkovich equation (Equation 16-39), or

\[ q_{Dd} = \frac{1}{1 + (t_a)_{Dd}} \quad (16-87) \]

with

\[ q_{Dd} = \left[ \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right] b_{pss} \quad (16-88) \]

\[ (t_a)_{Dd} = \left( \frac{m}{b_{pss}} \right) t_a \quad (16-89) \]

It must be noted that the \( q_{Dd} \) definition is now in terms of normalized pseudo-pressures, and the modified dimensionless decline time function, \( (t_a)_{Dd} \), is not in terms of real time but in terms of the material balance pseudo-time. Also notice that Equation 16-89 traces the path of a harmonic decline on the Fetkovich type curve with a hyperbolic exponent of \( b = 1 \).

However, there is a computational problem when applying Equation 16-82 because it requires the value of the average pressure \( \bar{p} \), which is itself a function of \( G \). Therefore the solution of Equation 16-83 is not direct and requires employing a numerical iterative method. The recommended solution procedure is based on a re-arranging of Equation 16-83 in the following familiar form of linear relationship:

\[ \frac{\bar{m}(p_i) - \bar{m}(p)}{q_g} = b_{pss} + m t_a \quad (16-90) \]

The iterative procedure for determining \( G \) and \( \bar{p} \) is shown in the following steps:
Step 1. Using the available gas properties, step up a table of $Z$, $\mu$, $p/Z$, $(p/Z\mu)$ versus $p$ for the gas system.

<table>
<thead>
<tr>
<th>Time</th>
<th>$p$</th>
<th>$Z$</th>
<th>$\mu$</th>
<th>$p/Z$</th>
<th>$(p/Z\mu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$p_0$</td>
<td>$Z_i$</td>
<td>$\mu_i$</td>
<td>$p_i/Z_i$</td>
<td>$p_i/(Z_i\mu_i)$</td>
</tr>
<tr>
<td>$\cdot$</td>
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</tr>
</tbody>
</table>

Step 2. Plot $(p/Z\mu)$ versus $p$ on a Cartesian scale and numerically determine the area under the curve for several values of $p$. Multiply each area by $(Z_i\mu_i/p_i)$ to give the normalized pseudo-pressure as follows:

$$\overline{m}(p) = \frac{\mu_i Z_i}{p_i} \int_{0}^{p} \left[ \frac{p}{\mu_i Z} \right] \, dp$$

The required calculations of this step can be performed in the following tabulated form:

<table>
<thead>
<tr>
<th>$p$</th>
<th>area</th>
<th>$\overline{m}(p) = (\text{area}) \frac{\mu_i Z_i}{p_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>$\cdot$</td>
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<td>$\cdot$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>$\cdot$</td>
<td>$\cdot$</td>
</tr>
</tbody>
</table>

Step 3. Make plots of $\overline{m}(p)$ and $p/Z$ versus $p$ on a Cartesian scale.

Step 4. Assume a value for the initial gas-in-place, $G$

Step 5. For each production data point of $G_p$ and $t$, calculate $\overline{p}/\overline{Z}$ from the gas material balance equation, Equation 16-21:

$$\frac{\overline{p}}{\overline{Z}} = \frac{p_i}{Z_i} \left( 1 - \frac{G_p}{G} \right)$$
Step 6. From the plot generated in Step 3, enter the graph of p versus p/Z with each value of the ratio p/Z and determine the value of the corresponding average reservoir pressure p. For each value of the average reservoir pressure p, determine the values m(p) for each p.

Step 7. For each production data point, calculate tₐ by applying Equation 16-82.

\[ tₐ = \left( \frac{\mu_c c_g}{q_i} \right) \frac{Z_i G}{2p_i} \left[ \bar{m}(p_i) - \bar{m}(\bar{p}) \right] \]

The calculation of tₐ can be conveniently performed in the following tabulated form:

<table>
<thead>
<tr>
<th>T</th>
<th>qᵢ</th>
<th>Gᵖ</th>
<th>p</th>
<th>m(p)</th>
<th>tₐ = (μ_c c_g) i \frac{Z_i G}{2p_i} [m(p_i) - m(\bar{p})]</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

Step 8. Based on the linear relationship given by Equation 16-91, plot [m(p_i) - m(\bar{p})]/q₉ versus tₐ on a Cartesian scale and determine the slope, m.

Step 9. Recalculate the initial gas-in-place, G, by using the value m from Step 8 and applying Equation 16-86 to give

\[ G = \frac{1}{cₐ} m \]

Step 10. The new value of G from Step 9 is used for the next iteration, i.e., starting from Step 5, and this process could continue until some convergence tolerance for G is met.

Palacio and Blasingame developed a modified Fetkovich-Carter type curve, as shown in Figure 16-21, to allow the performance of constant-rate
and constant-pressure gas flow solutions, the traditional Arps curve stems. To obtain a more accurate match to decline type curves than using flow-rate data alone, the authors introduced the following two complementary plotting functions:

- Integral function \((q_{Dd})_i\)

\[
(q_{Dd})_i = \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right) dt_a
\]  

(16-91)

- Derivative of the integral function \((q_{Dd})_{id}\)

\[
(q_{Dd})_{id} = \left( \frac{-1}{t_a} \right) \frac{d}{dt_a} \left[ \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right) dt_a \right]
\]  

(16-92)

Both functions can be easily generated by using simple numerical integration and differentiation methods.

To analyze gas-production data, the proposed method involves the following basic steps:

\[ \text{McCray Integral Type Curve} \]

\[ \text{McCray } q_{Dd} \text{ vs. } t_{Dd} \]

\[ \text{McCray } q_{Dd} \text{ vs. } t_{Dd} \]

\[ b = 0 \]

\[ b = 1 \]

Figure 16-21. Palacio-Blasingame type curve.
Step 1. Calculate the initial gas-in-place, G, as outlined previously.

Step 2. Construct the following table:

<table>
<thead>
<tr>
<th>t</th>
<th>(q_g)</th>
<th>(t_a)</th>
<th>(p_{wf})</th>
<th>(\bar{m}(p_i))</th>
<th>(\bar{m}(p_{wf}))</th>
<th>(\frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})})</th>
</tr>
</thead>
<tbody>
<tr>
<td>:</td>
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</tbody>
</table>

Plot \(q_g/[\bar{m}(p_i) - \bar{m}(p)]\) versus \(t_a\) on a Cartesian scale.

Step 3. Using the well production data as tabulated and plotted in Step 2, compute the two complementary plotting functions, as given by Equations (16-92) and (16-93) as a function of \(t_a\).

\[
(q_{Dd})_i = \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right) dt_a
\]

\[
(q_{Dd})_{id} = \left(-\frac{1}{t_a}\right) \frac{d}{dt_a} \left[ \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_g}{\bar{m}(p_i) - \bar{m}(p_{wf})} \right) dt_a \right]
\]

Step 4. Plot both functions, i.e., \((q_{Dd})_i\) and \((q_{Dd})_{id}\), versus \(t_a\) on a tracing paper so it can be laid over the type curve of Figure 16-22 for matching.

Step 5. Establish a match point and the corresponding dimensionless radius \(r_{Dd}\) value to confirm the final value of \(G\) and to determine other properties:

- \(G = \frac{1}{c_i} \left[ \frac{t_a}{t_{Dd}} \right]_{mp} \left[ \frac{(q_{Dd})_i}{q_{Dd}} \right]_{mp} \) \hspace{1cm} (16-93)
- \(A = \frac{5.615 \ G \ B_{gi}}{h \phi (1 - S_{wi})} \)
The authors used Fetkovich’s West Virginia gas well A, as given in Example 16-6, to demonstrate the use of the proposed type curve. The resulting fit of the data given in Example 16-6 by Palacio and Blasingame is shown in Figure 16-22.

4. Mattar and Anderson’s Flowing Material Balance

The flowing material balance method is a new technique that can be used to estimate the original gas-in-place (OGIP). The method, as introduced by Mattar and Anderson (2003), uses the concept of the normalized rate and material balance pseudo-time to create a simple linear plot, which extrapolates to fluids in place. The method uses the available production data in a manner similar to that of Palacio and Blasingame’s approach. The authors showed that for a depletion drive gas reservoir flowing under pseudo steady-state conditions, the flow system can be described by the following equation:
QN is the normalized cumulative production, as given by

\[
\frac{q}{m(p_i) - m(p_{wf})} = \frac{\Delta m(p)}{G b_{pss}} = \left(\frac{-1}{b_{pss}}\right) Q_N + \frac{1}{b_{pss}}
\]

Q_N is the normalized cumulative production, as given by

\[
Q_N = \frac{2q_i p_t t_a}{(c_1 \mu c_g) \Delta m(p)}
\]

And \(t_a\) is the Blasingame normalized material balance pseudo-time, as given by

\[
t_a = \frac{\mu c_g}{q_i} \frac{Z_i G}{2p_i} \left[\bar{m}(p_i) - \bar{m}(p)\right]
\]

The authors defined \(b_{pss}\) as the inverse productivity index, in psi^2/cp-MMscf, as follows:

\[
b_{pss} = 1.417 \times 10^6 \left(\ln\left(\frac{r_i}{r_{wa}}\right) - \frac{3}{4}\right)
\]
where \( p_i \) = initial pressure, psi
\( G \) = OGIP
\( r_e \) = drainage radius, ft
\( r_{wa} \) = apparent wellbore radius, ft

Thus, the previous expression suggests that a plot of \( q/\Delta m(p) \) versus 
\[ \frac{2q_p t_{act}}{(c_i \mu_i Z_i \Delta m(p))} \] on a Cartesian scale would produce a straight line with the following characteristics:

- x-axis intercept gives gas-in-place, \( G \)
- y-axis intercept gives \( b_{pss} \)
- Slope gives \(-1/G \ b_{pss}\)

The specific steps taken in estimating \( G \) are summarized below:

**Step 1.** Using the available gas properties, step up a table of \( Z, \mu, p/Z, \) 
\( (p/Z_\mu) \) versus \( p \) for the gas system.

**Step 2.** Plot \( (p/Z_\mu) \) versus \( p \) on a Cartesian scale and numerically determine the area under the curve for several values of \( p \) to give \( m(p) \) at each pressure.

**Step 3.** Assume a value for the initial gas-in-place, \( G \).

**Step 4.** Using the assumed value of \( G \) and for *each* production data point of \( G_p \) at time \( t \), calculate \( \bar{p}/Z \) from the gas material balance equation, Equation 16-21:

\[
\frac{\bar{p}}{Z} = \frac{p_i}{Z_i} \left( 1 - \frac{G_p}{G} \right)
\]

**Step 5.** For *each* production data point of \( q_t \) and \( t \), calculate \( t_a \) and the normalized cumulative production \( Q_N \):

\[
t_a = \frac{(\mu_e c_g) i}{q_i} \frac{Z_i G}{2p_i} \left[ \bar{m}(p_i) - \bar{m}(\bar{p}) \right]
\]

\[
Q_N = \frac{2q_t p_i t_a}{(c_i \mu_i Z_i) \Delta m(p)}
\]
Step 6. Plot \(\frac{q}{\Delta p}\) versus \(Q_N\) on a Cartesian scale and obtain the best line through the data points. Extrapolate the line to the x-axis and read the OGIP.

Step 7. The new value of \(G\) from Step 6 is used for the next iteration, i.e., Step 3, and this process could continue until some convergence tolerance for \(G\) is met.

5. Anash et al. Type Curves

The changes in gas properties can significantly affect reservoir performance during depletion; of utmost importance is the variation in the gas viscosity–compressibility product, \(\mu_g c_g\), which was ignored by Fetkovich in his development of his type curves. Anash et al. (2000) proposed three functional forms to describe the product, \(\mu_g c_t\), as a function of pressure. They conveniently expressed the pressure in a dimensionless form as generated from the gas material balance equation, to give

\[
\frac{p}{Z} = \frac{p_i}{Z_i} \left(1 - \frac{G_p}{G}\right)
\]

In a dimensionless form, the previous material balance equation is expressed as follows:

\[
p_D = (1 - G_{pd})
\]

where

\[
p_D = \frac{p/Z}{p_i/Z_i}
\]

\[
G_{pd} = \frac{G_p}{G}
\]

Anash and co-authors indicated that the product \((\mu_g c_t)\) can be expressed in a dimensionless ratio of \((\mu_g c_t)_i/(\mu_g c_t)\) as a function of the dimensionless pressure, \(p_D\), by one of the following three forms:

a) First-order polynomial

The first form is a first-degree polynomial that is adequate in describing the product, \(\mu_g c_t\), as a function of pressure at gas-reservoir pressures below
5,000 psi, that is, \( p_i < 5000 \). The polynomial is expressed in a dimensionless form as

\[
\frac{\mu_i \, c_u}{\mu \, c_i} = p_D
\]  

(16-96)

where

\[
\begin{align*}
c_{ti} &= \text{total system compressibility at } p_i, \text{ psi}^{-1} \\
\mu_i &= \text{gas viscosity at } p_i, \text{ cp}
\end{align*}
\]

b) Exponential model

The second form is adequate in describing the product, \( \mu_g c_t \), for high-pressure gas reservoirs, that is, \( p_i > 8000 \) psi.

\[
\frac{\mu_i \, c_u}{\mu \, c_i} = \beta_0 \exp(\beta_1 p_D)
\]  

(16-97)

i) General polynomial model

A third- or fourth-degree polynomial is considered by the authors a general model that is applicable to all gas-reservoir systems with any range of pressures, as given by

\[
\frac{\mu_i \, c_u}{\mu \, c_i} = a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4
\]  

(16-98)

The coefficient in Equations 16-98 and 16-99, \( \beta_0, \beta_1, a_0, a_1, \) etc., can be determined by plotting the dimensionless ratio \([\mu_i \, c_u/\mu \, c_i]\) versus \( p_D \) on a Cartesian scale, as shown in Figure 16-23, and using the least-squares regression model to determine the coefficients.

The authors also developed the following fundamental form of the stabilized gas flow equation:

\[
\frac{dG_p}{dt} = q_g = \frac{J_g}{c_u} \int_{p_{WD}}^{p_D} \left[ \frac{\mu_i \, c_u}{\mu \, c_i} \right] dp_D
\]

with the dimensionless bottom-hole flowing pressure defined as follows:

\[
p_{WD} = \frac{p_{wf}/Z_{wf}}{p_i/Z_i}
\]
where \( q_g \) = gas flow rate, scf/day
\( p_{wf} \) = flowing pressure, psia
\( Z_{wf} \) = gas deviation factor at \( p_{wf} \)
\( J_g \) = productivity index, scf/day, psia

Anash et al. presented their solutions in a type-curve format in terms of a set of the familiar dimensionless variables, \( q_Dd, t_{Dd}, r_{cD}, \) and a newly introduced correlating parameter, \( \beta \), that is a function of the dimensionless pressure. They presented three type-curve sets, as shown in Figures 16-24 through 16-26, one for each of the functional forms selected to
Figure 16-24. “First-order” polynomial solution for real-gas flow under boundary-dominated flow conditions. Solution assumes a μct profile that is linear with $P_D$. (Permission to copy by the SPE, 2000.)

Figure 16-25. “Exponential” solutions for real-gas flow under boundary-dominated flow conditions. (Permission to copy by the SPE, 2000.)
describe the product \( \mu c_t \) (i.e., first-order polynomial, exponential model, or general polynomial).

The methodology of employing the Anash et al. type curve is summarized in the following steps:

**Step 1.** Using the available gas properties, prepare a plot of \( (\mu_i c_t/\mu c_t) \) versus \( p_D \), where

\[
p_D = \frac{p/Z}{p_i/Z_i}
\]

**Step 2.** From the generated plot, select the appropriate functional form that describes the resulting curve:

- First-order polynomial

\[
\frac{\mu_i c_t}{\mu c_t} = p_D
\]
• Exponential model

\[ \frac{\mu_i c_{li}}{\mu c_i} = \beta_0 \exp(\beta_1 p_D) \]

• General polynomial model

\[ \frac{\mu_i c_{li}}{\mu c_i} = a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4 \]

Use a regression model (i.e., least-squares) to determine the coefficient of the selected functional form that adequately describes \( \frac{\mu_i c_{li}}{\mu c_i} \) versus \( p_D \).

**Step 3.** Plot the historical flow rate, \( q_g \), versus time, \( t \), on a log-log scale with the same logarithmic cycles as the one given by the selected type curves (i.e., Figures 16-24 through 16-26).

**Step 4.** Using the type-curve matching technique described previously, select a match point and record

- \( (q_g)_{mp} \) and \( (q_Dd)_{mp} \)
- \( (t)_{mp} \) and \( (t_{Dd})_{mp} \)
- \( (r_o)_{mp} \)

**Step 5.** Calculate the dimensionless pressure \( p_{WD} \) using the bottom-hole flowing pressure,

\[ p_{WD} = \frac{p_{wf}}{Z_{wf}} \frac{p_i}{Z_i} \]  

(16-99)

**Step 6.** Depending on the selected functional form in Step 2, calculate the constant \( \alpha \) for the selected functional model:

- For the first-order polynomial

\[ \alpha = \frac{1}{2} (1 - p_{WD}^2) \]  

(16-100)
For the exponential model
\[ \alpha = \frac{\beta_0}{\beta_1} \left[ \exp(\beta_1) - \exp(\beta_1 p_{wD}) \right] \]  
(16-101)

where \( \beta_0 \) and \( \beta_1 \) are the coefficients of the exponential model.

For the polynomial function (assuming a fourth-degree polynomial)
\[ \alpha = A_o + A_1 + A_2 + A_3 + A_4 \]  
(16-102)

where
\[ A_o = -(A_1 p_{wD} + A_2 p_{wD}^2 + A_3 p_{wD}^3 + A_4 p_{wD}^4) \]
\[ A_1 = a_o \]
\[ A_2 = \frac{a_1}{2} \]
\[ A_3 = \frac{a_2}{3} \]
\[ A_4 = \frac{a_3}{4} \]

**Step 7.** Calculate the well **productivity index**, \( J_g \), in scf/day – psia, by using the **flow-rate match point** and the constant \( \alpha \) of Step 6 in the following relation:
\[ J_g = \frac{C_{ni}}{\alpha} \left( \frac{q_g}{q_{Dd}} \right)_{mp} \]  
(16-103)

**Step 8.** Estimate the OGIP, in scf, from the **time match point**:
\[ G = \frac{J_g}{C_{ni}} \left( \frac{t}{t_{Dd}} \right)_{mp} \]  
(16-104)
Step 9. Calculate the reservoir drainage area, \( A \), in \( \text{ft}^2 \), from the following expression:

\[
A = \frac{5.615 B_{gi} G}{\phi h (1 - S_{wi})}
\]  
\hspace{1cm} (16-105)

where  
\( A \) = drainage area, \( \text{ft}^2 \)  
\( B_{gi} \) = gas formation volume factor at \( p_i \), bbl/scf  
\( S_{wi} \) = connate-water saturation

Step 10. Calculate the permeability, \( k \), in md, from the match curve of the dimensionless drainage radius, \( r_e D \):

\[
k = \frac{141.2 \mu_i B_{gi} J_g}{h} \left( \ln[r_e D]_{mp} - \frac{1}{2} \right)
\]  
\hspace{1cm} (16-106)

Step 11. Calculate the skin factor from the following relationships:

- Drainage radius  
  \[
r_e = \sqrt{\frac{A}{\pi}}
\]  
  \hspace{1cm} (16-107)

- Apparent wellbore radius  
  \[
r_{wa} = \frac{r_e}{(r_e D)_{mp}}
\]  
  \hspace{1cm} (16-108)

- Skin factor  
  \[
s = -\ln \left( \frac{r_{wa}}{r_w} \right)
\]  
  \hspace{1cm} (16-109)

Example 16-10

The West Virginia gas well, \( A \), is a vertical gas well that has been hydraulically fractured and is undergoing depletion. The production data were presented by Fetkovich (1980) and used in Example 16-6. A summary of the reservoir and fluid properties is given below:

\( r_w = 0.354 \text{ ft} \)  
\( h = 70 \text{ ft} \)  
\( \phi = 0.06 \)
Solution

Step 1. Figure 16-27 shows the type-curve match of the production data with that of Figure 16-24, to give:

\[(q_{Dd})_{mp} = 1.0\]

\[(q_g)_{mp} = 1.98 \times 10^6 \text{ scf/day}\]

\[(t_{Dd})_{mp} = 1.0\]

\[(t)_{mp} = 695 \text{ days}\]

\[r_{cD} = 28\]

Step 2. Calculate the productivity index from Equation 16-104:

\[J_g = \frac{C_d}{\alpha} \left( \frac{q_g}{q_{Dd}} \right)_{mp}\]

\[J_g = \frac{0.000184}{0.4855} \left( \frac{1.98 \times 10^6}{1.0} \right) = 743.758 \text{ $/cf/day - psi}$\]
Step 3. Solve from G by applying Equation 16-105:

\[
G = \frac{J_{g}}{C_{ii}} \left( \frac{t}{t_{Dd}} \right)^{mp}
\]

\[
G = \frac{743.758}{0.0001824} \left( \frac{695}{1.0} \right) = 2.834 \beta \text{scf}
\]

Step 4. Calculate the drainage area from Equation 16-106:

\[
A = \frac{5.615 B_{g} G}{\phi h (1 - S_{wi})}
\]

\[
A = \frac{5.615 \times (0.00071) \times (2.834 \times 10^{9})}{(0.06) \times (70) \times (1 - 0.35)} = 4.1398 \times 10^{6} \text{ ft}^{2} = 95 \text{ acres}
\]

Step 5. Compute the permeability from the match on the \( r_{cD} = 28 \) transient stem by using Equation 16-107:
**Step 6.** Calculate the skin factor by applying Equations 16-108 and 16-109:

\[
k = \frac{(141.2) (0.0225) (0.00071) (743.76)}{70} \left( \ln(28) - \frac{1}{2} \right)
= 0.0679 \text{ md}
\]

6. **Decline-Curve Analysis for Fractured Wells**

A fracture is defined as a single crack initiated from the wellbore by hydraulic fracturing. It should be noted that fractures are different from “fissures,” which are the formation of natural fractures. Hydraulically induced fractures are usually vertical, but can be horizontal if the formation is less than about 3,000 ft deep. Vertical fractures are characterized by the following properties:

- Fracture half-length \( x_f \), in ft
- Dimensionless radius \( r_{eD} \), where \( r_{eD} = r_e / x_f \)
- Fracture height \( h_f \), which is often assumed equal to the formation thickness, in ft
- Fracture permeability \( k_f \), in md
- Fracture width \( w_f \), in ft
- Fracture conductivity \( F_C \), where \( F_C = k_f w_f \)

The analysis of fractured-well tests deals with the identification of well and reservoir variables that would have an impact on future well performance. However, fractured wells are substantially more complicated. The well-penetrating fracture has unknown geometric features, that is, \( x_f \), \( w_f \), and \( h_f \), and unknown conductivity properties.
Many authors have proposed three transient flow models to consider when analyzing transient pressure data from vertically fractured wells; these are as follows:

- Infinite-conductivity vertical fractures
- Finite-conductivity vertical fractures
- Uniform-flux fractures

Description of these three types of fractures are as follows:

**Infinite-Conductivity Vertical Fractures**

These fractures are created by conventional hydraulic fracturing and characterized by a very high conductivity, which, for all practical purposes, can be considered infinite. In this case, the fracture acts similarly to a large-diameter pipe with infinite permeability and, therefore, there is essentially no pressure drop from the tip of the fracture to the wellbore, that is, no pressure loss in the fracture. This model assumes that the flow into the wellbore is only through the fracture and exhibits three flow periods:

- Fracture linear flow period
- Formation linear flow period
- Infinite-acting pseudo-radial flow period

Several specialized plots are used to identify the start and end of each flow period. For example, an early time log-log plot of Δp versus Δt will exhibit a straight line of half-unit slope. These flow periods associated with infinite conductivity fractures and the diagnostic specialized plots will be discussed later in this section.

**Finite-Conductivity Vertical Fractures**

These are very long fractures created by massive hydraulic fracture (MHF). These types of fractures need large quantities of propping agent to maintain them open, and, as a result, the fracture permeability, k_f, is lower than that of the infinite-conductivity fractures. These finite-conductivity vertical fractures are characterized by measurable pressure drops in the fracture and, therefore, exhibit unique pressure responses during testing of hydraulically fractured wells. The transient pressure behavior for this system can include the following four sequence flow periods (to be discussed later):
• Initially, linear flow within the fracture
• Next, bilinear flow
• Then, linear flow in the formation
• And eventually, infinite acting pseudo-radial flow

Uniform-Flux Fractures

A uniform flux fracture is one in which the reservoir fluid-flow rate from the formation into the fracture is uniform along the entire fracture length. This model is similar to the infinite-conductivity vertical fracture in several aspects. The difference between these two systems occurs at the boundary of the fracture. The system is characterized by a variable pressure along the fracture and exhibits essentially two flow periods:

• Linear flow
• Infinite-acting pseudo-radial flow

Except for highly propped and conductive fractures, it is thought that the uniform-influx fracture theory better represents reality than the infinite-conductivity fracture; however, the difference between the two is rather small.

The fracture has a much greater permeability than the formation it penetrates; hence, it influences the pressure response of a well test significantly. The general solution for the pressure behavior in a reservoir is expressed in terms of dimensionless variables. The following dimensionless groups are used when analyzing pressure transient data in a hydraulically fractured well:

• Conductivity group: \( F_{CD} = \frac{k_f}{k} \frac{w_f}{x_f} = \frac{F_C}{k x_f} \)

• Fracture group: \( r_{CD} = \frac{r}{x_f} \)

where \( x_f \) = fracture half-length, ft
\( w_f \) = fracture width, ft
\( k_f \) = fracture permeability, md
\( k \) = pre-frac formation permeability, md
\( F_C \) = fracture conductivity, md-ft
\( F_{CD} \) = dimensionless fracture conductivity
Pratikno, Rushing, and Blasingame (2003) developed a new set of type curves specifically for finite-conductivity vertically fractured wells centered in bounded circular reservoirs. The authors used analytical solutions to develop these type curves and to establish a relation for the decline variables.

Recall that the general dimensionless pressure equation for a bounded reservoir during pseudosteady-state flow is given by Equation 6-137:

\[ p_D = 2\pi t_{DA} + \left( \frac{1}{2} \ln \left( \frac{A}{r_w^2} \right) + \frac{1}{2} \ln \left( \frac{2.2458}{C_A} \right) \right) + s \]

with the dimensionless time based on the wellbore radius, \( t_D \), or drainage area, \( t_{DA} \), as given by Equations 6-87 and 6-87a:

\[ t_D = \frac{0.0002637 k t}{\phi \mu c_i r_w^2} \]
\[ t_{DA} = \frac{0.0002637 k t}{\phi \mu c_i A} = t_A \left( \frac{r_w^2}{A} \right) \]

The authors adopted the last form and suggested that, for a well producing under pseudosteady-state at a constant rate with a finite-conductivity fracture in a circular reservoir, the dimensionless pressure drop can be expressed as follows:

\[ p_D = 2\pi t_{DA} + b_{Dpss} \]

or

\[ b_{Dpss} = p_D - 2\pi t_{DA} \]

where the term \( b_{Dpss} \) is the dimensionless pseudosteady-state constant that is independent of time; however, \( b_{Dpss} \) is a function of

- the dimensionless radius, \( r_{eD} \), and
- the dimensionless fracture conductivity, \( F_{CD} \)

The authors note that, during pseudosteady flow, the equation describing the flow during this period yields constant values for given values of \( r_{eD} \) and \( F_{CD} \) that are closely given by the following relationship:

\[ b_{Dpss} = \ln(r_{eD}) - 0.049298 + \frac{0.43464}{r_{eD}^2} + \frac{a_1 + a_2 u + a_3 u^2 + a_4 u^3 + a_5 u^4}{1 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4} \]
with

\[ \text{u} = \ln(\text{FCD}) \]

where

\[ a_1 = 0.93626800, \quad b_1 = -0.38553900 \]
\[ a_2 = -1.00489000, \quad b_2 = -0.06988650 \]
\[ a_3 = 0.31973300, \quad b_3 = -0.04846530 \]
\[ a_4 = -0.04235320, \quad b_4 = -0.00813558 \]
\[ a_5 = 0.00221799 \]

Based on the above equations, Pratikno et al. (2003) used Palacio and Blasingame’s previously defined functions (i.e., \( t_a \), \( q_{\text{Dd}} \), and \( q_{\text{Ddi}} \)) and the parameters \( r_{\text{ed}} \) and \( \text{FCD} \) to generate a set of decline curves for a sequence of 13 values for \( \text{FCD} \) with a sampling of \( r_{\text{ed}} = 2, 3, 4, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, \) and 1,000. Type curves for \( \text{FCD} \) of 0.1, 1, 10, 100, and 1,000 are shown in Figures 16-28 through 16-32.

The authors recommend the following type-curve matching procedure, which is similar to the methodology used in applying Palacio and Blasingame’s type curve:

**Figure 16-28.** Fetkovich-McCray decline type curve–rate versus material balance time format for a well with a finite conductivity vertical fracture (\( \text{FCD}=0.1 \)). (Permission to copy by the SPE, 2003.)
Fetkovich-McCray Type Curve for a Vertical Well with a Finite Conductivity Vertical Fracture ($F_{cD} = 1$)

**Figure 16-29.** Fetkovich-McCray decline type curve–rate versus material balance time format for a well with a finite conductivity vertical fracture ($F_{cD} = 1$). (Permission to copy by the SPE, 2003.)

Fetkovich-McCray Type Curve for a Vertical Well with a Finite Conductivity Vertical Fracture ($F_{cD} = 10$)

**Figure 16-30.** Fetkovich-McCray decline type curve–rate versus material balance time format for a well with a finite conductivity vertical fracture ($F_{cD} = 10$). (Permission to copy by the SPE, 2003.)
Figure 16-31. Fetkovich-McCray decline type curve–rate versus material balance time format for a well with a finite conductivity vertical fracture (FcD=100). (Permission to copy by the SPE, 2003.)

Figure 16-32. Fetkovich-McCray decline type curve–rate versus material balance time format for a well with a finite conductivity vertical fracture (FcD=1000). (Permission to copy by the SPE, 2003.)
**Step 1.** Calculate the dimensionless fracture conductivity, \( F_{CD} \), and the fracture half-length, \( x_f \).

**Step 2.** Assemble the available well data in terms of bottom-hole pressure and the flow rate, \( q_t \) (in STB/day for oil or Mscf/day for gas) as a function of time. Calculate the **material balance pseudotime**, \( t_a \), for each given data point by using the following equations:

- For oil:  
  \[
  t_a = \frac{N_p}{q_t}
  \]

- For gas:  
  \[
  t_a = \frac{\left(\mu_g c_g\right)_i Z_i G}{2p_i} \left[\overline{m}(p_i) - \overline{m}(\overline{p})\right]
  \]

where \( \overline{m}(p_i) \) and \( \overline{m}(p) \) are the normalized pseudo-pressures, as defined by Equations 16-84 and 16-85:

\[
\overline{m}(p_i) = \frac{\mu_g Z_i}{p_i} \int_0^p \frac{p}{\mu_g Z} \, dp
\]

\[
\overline{m}(p) = \frac{\mu_g Z_i}{p_i} \int_0^p \frac{p}{\mu_g Z} \, dp
\]

Notice that the GOIP must be calculated iteratively, as illustrated previously by Palacio and Blasingame (1993).

**Step 3.** Using the well-production data tabulated and plotted in Step 2, compute the following three complementary plotting functions:

- Pressure drop normalized rate, \( q_{Dd} \)
- Pressure drop normalized rate integral function, \( (q_{Dd})_{i} \)
- Pressure drop normalized rate integral–derivative function, \( (q_{Dd})_{id} \)

For gas:

\[
q_{Dd} = \frac{q_g}{\overline{m}(p_i) - \overline{m}(p_{wf})}
\]

\[
(q_{Dd})_i = \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_g}{\overline{m}(p_i) - \overline{m}(p_{wf})} \right) \, dt_a
\]
• For oil:

\[
(q_{Dd})_o = \frac{q_o}{p_i - p_{wf}}
\]

\[
(q_{Dd})_i = \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_o}{p_i - p_{wf}} \right) dt_a
\]

\[
(q_{Dd})_{id} = \frac{-1}{t_a} \frac{d}{dt_a} \left[ \frac{1}{t_a} \int_0^{t_a} \left( \frac{q_o}{p_i - p_{wf}} \right) dt_a \right]
\]

Step 4. Plot the three gas or oil functions, \((q_{Dd})_i\), \((q_{Dd})_{id}\), and \((q_{Dd})_{id}\), versus \(t_a\) on a tracing paper so that it can be laid over the type curve with the appropriate value of \(F_{CD}\).

Step 5. Establish a match point for each of the three functions \((q_{Dd})_i\), \((q_{Dd})_{id}\), and \((q_{Dd})_{id}\). Once a match is obtained, record the time and rate match points as well as the dimensionless radius value, \(r_{eD}\):

a) Rate-axis match point Any \((q/\Delta p)_{MP} - (q_{Dd})_{MP}\) pair

b) Time-axis match point Any \((t)_{MP} - (t_{Dd})_{MP}\) pair

c) Transient flow stem Select the \((q/\Delta p)_i\), \((q/\Delta p)_{id}\) functions that best match the transient data stem and record \(r_{eD}\).

Step 6. Solve for \(b_{Dpss}\) by using the values of \(F_{CD}\) and \(r_{eD}\):

\[ u = \ln(F_{CD}) \]

\[ b_{Dpss} = \ln(r_{eD}) - 0.049298 + \frac{0.43464}{r_{eD}^2} + \frac{a_1 + a_2 u + a_3 u^2 + a_4 u^3 + a_5 u^4}{1 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4} \]

Step 7. Using the results of the match point, estimate the following reservoir properties:
• For gas: 
\[ G = \frac{1}{c_u} \left[ \frac{t_a}{t_{Dd}} \right]_{mp} \left[ \frac{(q_g/\Delta m(p))}{q_{Dd}} \right]_{mp} \]

\[ k_g = \frac{141.2 B_{gi} \mu_{gi}}{h} \left[ \frac{(q_g/\Delta m(p))_{MP}}{(q_{Dd})_{MP}} \right] b_{Dpss} \]

\[ A = \frac{5.615 G B_{gi}}{h \phi (1 - S_{wi})} \]

\[ r_c = \sqrt{\frac{A}{\pi}} \]

• For oil: 
\[ N = \frac{1}{c_t} \left[ \frac{t_a}{t_{Dd}} \right]_{mp} \left[ \frac{(q_o/\Delta p)}{q_{Dd}} \right]_{mp} \]

\[ k_o = \frac{141.2 B_{oi} \mu_{goi}}{h} \left[ \frac{(q_o/\Delta p)_{MP}}{(q_{Dd})_{MP}} \right] b_{Dpss} \]

\[ A = \frac{5.615 N B_{si}}{h \phi (1 - S_{wi})} \]

\[ r_c = \sqrt{\frac{A}{\pi}} \]

where 
- \( G \) = gas-in-place, Mscf
- \( N \) = oil-in-place, STB
- \( B_{gi} \) = gas formation volume factor at \( p_i \), bbl/Mscf
- \( A \) = drainage area, ft²
- \( r_c \) = drainage radius, ft
- \( S_{wi} \) = connate-water saturation

**Step 8.** Calculate the fracture half-length, \( x_f \), and compare with Step 1:

\[ x_f = \frac{r_c}{r_{eD}} \]
Example 16-11

A Texas field vertical gas well has been hydraulically fractured and is undergoing depletion. A summary of the reservoir and fluid properties is as follows:

\[
\begin{align*}
\text{\(r_w\)} &= 0.333 \text{ ft} \\
\text{\(h\)} &= 170 \text{ ft} \\
\phi &= 0.088 \\
T &= 300^\circ\text{F} \\
\gamma_g &= 0.70 \\
B_{gi} &= 0.5498 \text{ bbl/Mscf} \\
\mu_{gi} &= 0.0361 \text{ cp} \\
c_i &= 5.1032 \times 10^{-5} \text{ psi}^{-1} \\
p_i &= 9330 \text{ psia} \\
p_{wf} &= 710 \text{ psia} \\
S_{wi} &= 0.131 \\
F_{CD} &= 5.0
\end{align*}
\]

Fetkovich-McCray Type Curve for a Vertical Well with a Finite Conductivity Vertical Fracture (\(F_{CD} = 5\))

[Example 1-Low Permeability/High Pressure Gas Reservoir (Texas)]

Figure 16-33. Match of production data for Example 1 on the Fetkovich-McCray decline type curve (pseudo-pressure drop normalized rate versus material balance time format) for a well with a finite conductivity vertical fracture (\(F_{CD}=5\)). (Permission to copy the SPE, 2003.)
Figure 16-33 shows the type-curve match for $F_{CD} = 5$, with the matching points:

- $(q_{Dd})_{mp} = 1.0$
- $[(q_g/\Delta m(\bar{p}))]_{mp} = 0.89 \text{ Mscf/psi}$
- $(t_{Dd})_{mp} = 1.0$
- $(t_a)_{mp} = 58 \text{ days}$
- $(r_{eD})_{mp} = 2.0$

Perform type-curve analysis on this gas well.

**Solution**

**Step 1.** Solve for $b_{Dpss}$ by using the values of $F_{CD}$ and $r_{eD}$:

- $u = \ln(F_{CD}) = \ln(5)1.60944$

$$b_{Dpss} = \ln(r_{eD}) - 0.049298 + \frac{0.43464}{r_{eD}^2} + \frac{a_1 + a_2 u + a_3 u^2 + a_4 u^3 + a_5 u^4}{1 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4}$$

$$b_{Dpss} = \ln(2) - 0.049298 + \frac{0.43464}{2^2} + \frac{a_1 + a_2 u + a_3 u^2 + a_4 u^3 + a_5 u^4}{1 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4} = 1.0022$$

**Step 2.** Using the results of the match point, estimate the following reservoir properties:

$$G = \frac{1}{c_t} \left[ \frac{t_a}{t_{Dd}} \right]_{mp} \left[ \frac{(q_g/\Delta m(\bar{p}))}{q_{Dd}} \right]_{mp}$$

$$G = \frac{1}{5.1032 \times 10^{-5}} \left[ \frac{58}{1.0} \right]_{mp} \left[ \frac{0.89}{1.0} \right] = 1.012 \times 10^6 \text{ MMscf}$$

$$k_g = \frac{141.2 B_{gi} \mu_{gi}}{h} \left[ \frac{(q_g/\Delta m(\bar{p}))_{MP}}{(q_{Dd})_{MP}} \right] b_{Dpss}$$
Step 3. Calculate the fracture half-length, \( x_f \), and compare with Step 1:

\[ x_f = \frac{r_e}{2} = \frac{277}{2} = 138 \text{ ft} \]

**PROBLEMS**

1. A gas well has the following production history:

<table>
<thead>
<tr>
<th>Date</th>
<th>Time, months</th>
<th>( q_t ), MMscf/month</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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</tr>
<tr>
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<td>1</td>
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<td>941</td>
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<td>905</td>
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</tr>
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<td>7/1/2000</td>
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</tr>
<tr>
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<td>10/1/2000</td>
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</tr>
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<td>641</td>
</tr>
</tbody>
</table>
a) Use the first six months of the production history data to determine the coefficient of the decline-curve equation.
c) Assuming that the economic limit is 20 MMscf/month, estimate the time to reach the economic limit and the corresponding cumulative gas production.

2. The volumetric calculations on a gas well show that the ultimate recoverable reserves, \( G_{pa} \), are 18 MMMscf of gas. By analogy with other wells in the area, the following data are assigned to the well:

- Exponential decline
- Allowable (restricted) production rate = 425 MMscf/month
- Economic limit = 20 MMscf/month
- Nominal decline rate = 0.034 month\(^{-1}\)

Calculate the yearly production performance of the well.

3. The following data are available on a gas well’s production:

\[
\begin{align*}
\pi_l &= 4100 \text{ psia} \\
\phi &= 0.10 \\
p_{wf} &= 400 \text{ psi} \\
S_{wi} &= 0.30 \\
T &= 600°R \\
\gamma_g &= 0.65 \\
h &= 40 \text{ ft}
\end{align*}
\]

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</thead>
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</table>

Calculate the GOIP and the drainage area.
REFERENCES


